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Modelling Efficacy of Probability Density Functions (PDF) in Wireless Communication

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Abstract- ***Probability Density Function (PDF) has been one of the major statistical functions employed in modelling of wireless communication signal. Its versatility gains wide popularity and the importance of this distribution cannot be over emphasized. However, the PDF statistical models are always in complex integrals thereby, truncating its immediate use. This was overcome by the use of Moment Generating Function (MGF) over combined Rayleigh and Rician fading channels. PDF methodology was employed to study the behavior of the signal propagated in wireless media and evaluated using Bit Error Rate (BER) and Outage probability (P_{out}). The Moment Generating Function (MGF) developed was an infinite series, which was truncated using Padé Approximation (PA). This was used to derive the PDF of the cascaded Rayleigh-Rician fading distribution. The results showed that the PDF model is effective in predicting the behavior of wireless signals. This, further revealed that the higher the SNR, the lower the BER and the higher the threshold value of the signal, the less the Outage probability and the better the signal reception. Simulation results agreed with the theoretical results obtained from the developed expressions.***

Keywords: *Bit Error Rate, Moment Generating Function, Outage Probability,* Padé*Approximation, Probability Density Function, Rayleigh-Rician fading distribution.*

1.0 Introduction

Statistical function has been a major tool in theoretical physics and referred to as statistical Physics. Invariably from time immemorial, it has been a powerful and reliable tool in analyzing communication channel. Since [1] first used Entropy as a scientific concept in thermodynamics and the eventual introduction of entropy by [2] to problems of coding theory and data transmission, there has been a growing interest in statistical analysis of Wireless Communication System (WCS). In [27] work was carried out on Entropy and Methods of Moments in evaluation of error probability in digital communication, and confirm its application in WCS. The work was based on the earlier work of [3], [4]. The effort of these three authors illustrated the relationship between maximum entropy, conditional probability and moments thereby leading to many research outcomes on the use of statistical functions in wireless communication modeling.

The modelling of WCS system is categorized into three; empirical modelling, deterministic modelling and stochastic modelling. The empirical model is based on data collected at certain location and therefore localized and may not be suitable for other location. Some of the empirical models include but not exhaustive, Ericson model, COST-231 model, Hata model and Okumura model. The deterministic model relies on Electromagnetic Wave (EMW) equations to analyze the performance of the signal transmitted in the medium. The stochastic model makes use of statistical distributions and parameters to analyze the WCS. This approach treats signal transmitted in the WC medium as a random variable. Statistical distributions used in modelling WC channel includes but not limited to; Rayleigh, Rician and Nakagami distribution [5].

Wireless communication is characterized by multipath fading effects. In mobile satellite communication, the mobile radio channel undergoes many time varying signal impairments such as multipath fading, shadowing, ionospheric scintillation, free space propagation loss, and thermal noise.As a result of the distinctiveness and unpredictable nature of wireless systems, the transmitted signal could be degraded by scintillation fading through the ionosphere and terrestrial multipath fading in the troposphere, wireless system is thus constrained by the channel fading. Mobile radio propagation channel introduces fundamental limitations on the performance of any wireless communication. Fading refers to the time variation of a received signal power caused by changes in the transmission medium; mitigating the delirious effect of multipath propagation is a major continuous exercise of researchers, in order to achieve a near perfect reception of the transmitted signal. Many methods have been proposed and used; those methods fall into three categories; firstly, Power control, this involves varying the Effective Isotropic Radiation Power (EIRP) of the signal, to enhance the Carrier to Noise Ratio (CNR). Secondly, signal processing, is a method by which the parameters of a signal are changed to improve the performance. Diversity is the last in which the advantage of multipath propagation is exploited by choosing a different path or time to take advantage of de-correlated fading [6]. Diversity involves transmitting a copy of signal through multiple paths in time, frequency, polarizations or antennas. This is due to the fact that not all signal transmitted or scattered will be in deep fade. Diversity is a technique used to compensate fast fading and can be implemented using space, antenna, angle, polarization and so on. Whatever the type of diversity used, the combining method falls under three categories namely, Maximal Ratio Combining (MRC), Equal Gain Combining (EGC), Selection Combining (SC).

In [7] investigation was carried out on the impact of channel estimation error on the performance of MRC system in the presence of Co-Channel Interference (CCI) with an arbitrary Power Interference to Noise Ratio (PINR). The subjectivity of WCS to additive channel noise, multipath fading and CCI were identified as major factors that affect performance in this channel. The work derived exact closed-form expressions of the P_{out} for MRC systems in Rayleigh fading channels with channel estimation error in the presence of CCI, where the power level of CCI could be arbitrary. The P_{out} was derived through the PDF of the output Signal to Interference Noise Ratio (SINR) and was in the form of finite sums which was easily evaluated numerically. The Average Bit Error Probability (ABEP) was developed via the same approach and the closed-form expression was found to be in terms of the confluent hypergeometric function of the second order. The result was theoretically evaluated and simulation results were provided to demonstrate the accuracy of the analytical results. It was concluded that the power gain loss due to the channel estimation error can be reduced using diversity technique.

In [8], there was a presentation on the performance analysis of MRC receiver with channel estimation error and CCI in Nakagami-m fading channel. The author identified Phase Error (PE) and CCI as a major drawback in wireless mobile communication. The author presented diversity as a major solution to the problem. The research, theoretically modelled the mobile wireless communication as a Nakagami-m fading channel. By employing PDF approach, new closed-form expressions were obtained for both P_{out} and Average Symbol Error Probability. The models were validated using simulation, the model derived showed that diversity combining techniques can effectively solve the problems identified. In [9], there was a derivation of a stochastic model to study L-branch EGC combiner with carrier PE and CCI over Nakagami-m fading channel using the PDF method. The author made use of the statistical formula to derive the PDF model for the joint modelling of PE and CCI. The results showed that the PE and CCI have a degrading effect on the outage probability and ABEP. The author concluded that the expressions derived were effective in predicting the effect of both the PE and CCI on signal transmitted through a wireless medium. The Nakagami-model is not effective in modelling terrestrial environment where the buildings and other structures are clustered together which unavoidably block Line of Sight (LOS) component.

In [10], development of L-MRC receiver (where L is the number of paths) with estimation error over Hoyt fading channels was carried out. The authors used the PDF based approach combined with the conditional BER of the Hoyt fading channel. Channel capacity and ABEP were used as performance metrics. New closed-form expressions were obtained for channel capacity and ABER through integral analysis of the model equation. The current attempt is to demonstrate the workability of the PDF model developed through the MGF in a cascaded Rayleigh-Rician fading channel over EGC diversity.

2.0 Statistical Functions in Wireless Communication

There are about five important statistical parameters used to represent the fading distribution and very relevant to this work, these are enumerated as follows:

2.1 Expectations (Expected Value)

The expected value known as expectations is the mean value of a Random Variable X, and is defined by [11], [12], [13], [14] as

$$E\{X\} = \int_{-\infty}^{\infty} x P_x (x) dx \quad (1)$$

where $E\{X\}$ is called the expected value operator.

x is a random variable

$P_x(x)$ is the PDF of the random variable

This expectation as defined above is regarded as the weighted average of the value of X [15]. The n^{th} moment of a probability distribution of a Random Variable (RV), X, $E\{X^n\}$, is given as

$$E\{X^n\} = \int_{-\infty}^{\infty} x^n P_x(x)dx \tag{2}$$

Sklar (2002) stated that for the purpose of communication system analysis, the most important moments of 'X' are the first two moments. Therefore, when $n = 1$ (2) gives the mean value of RV, whereas $n = 2$ gives the mean square value of x, as follows:

$$E\{X^2\} = \int_{-\infty}^{\infty} x^2 P_x(x)dx \tag{3}$$

The variance is the average of the sum of the deviation from the mean that is

$$\sigma^2 = E\{(X - m_x)^2\} = \int_{-\infty}^{\infty} (x - m_x)^2 P_x(x)dx \tag{4}$$

Where, σ^2 is the variance of the distribution.

m_x is the mean of the distribution.

Using the properties of the expected value and expanding equation (4), it becomes

$$\sigma^2 = E\{(X - m_x)^2\} = E\{X^2\} - m_x^2 \tag{5}$$

The variance is therefore, the difference between the mean square value and the square of the mean. The expected value or expectation is therefore known as statistical averages. The n^{th} moment of the Rayleigh fading distribution is expressed by [16] as

$$E(\beta_l^n) = \Gamma(1+n)\bar{\beta}_l^n \tag{6}$$

The moment of the Rician fading distribution is given by Simon and Alouini, (2005) as

$$E(\beta_l^n) = \frac{\Gamma(1+n)}{(k_l+1)^n} \; {}_1F_1(-n; 1; -k_l)\bar{\beta}_l^n \tag{7}$$

Where, $n \in \{1,2,3 \dots\dots\dots\}$ that is, set of natural numbers

$\bar{\beta}_l$ is the average signal to noise ratio of the l^{th} path

k_l is the LOS of the l^{th} diversity branch

$\Gamma(.)$ is the complete gamma function

${}_1F_1(.;.;.)$ is the generalized hyper geometrical function

the expressions for $\Gamma(.)$ and ${}_1F_1(.;.;.)$ are defined inm [17],[18] as

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} \tag{8}$$

$${}_1F_1(a;b;c) = \sum_{k=0}^{\infty} \frac{(a_1)_k}{(b_1)_k} c^k \tag{9}$$

where $(.)_k$ is the rising factorial or Pochhammersymbol defined as

$$(a)_k = \frac{\Gamma(a+1)}{\Gamma(a)} = a(a+1)(a+2)\dots\dots(a+k-1) \text{ where } (a)_0 = 1 \tag{10}$$

2.2 Cumulative Distribution Functions

The Cumulative Distribution Function (CDF) of a Random Variable (RV) $X(k)$ evaluated at any fixed value of 'X' is defined as the probability that the RV 'X' takes a value less than or equal to x and is expressed by [19], [13], as

$$F(x) = P[X(k) \le x] \tag{11}$$

This is an important distribution in WCS, and it actually denotes the P_{out}. Outage probability of WCS is a fundamental problem in communication theory and it is one of the most widely studied in literature, [20]

2.3 Probability Density Function

Probability Density Function (PDF) describes the possibilities of a RV to take on a given value, which is the relative likelihood of a RV signal to exist at a given point. In WCS theory, PDF is obtained by differentiating $F(x)$ in (11)

$$P_x(x) = \frac{d}{dx}(F(x)) \quad (12)$$

Where $F(x)$ depends on the distribution being considered likewise the result of the differential. The PDF is of utmost important because it can be used to derive Moment Generating Function (MGF) [28]. In statistic communication theory, this is an important parameter which is used to find the Bit Error Rate (BER), amount of fading, outage probability and also related to other performance metrics

2.4 Moment Generating Function

MGF is a compliment of PDF and has been known to be a technique to provide another method of representing a probability distribution by means of a certain function of a single variable. This powerful tool is the Laplace transform of the PDF and expressed by [21], [22], [16] as

$$M_\beta(s) = E\{X^{s\beta}\} = \int_0^\infty e^{s\beta} P_\beta(\beta)\, d\beta\beta > 0$$

(13)

where s is a variable and the value of which is chosen to ensure the convergence of the integral, and $P_\beta(\beta)$ is the PDF of β. MGF determines the distribution that is if RVs X and Y have the same MGF then X and Y have the sameCumulative Density Function (CDF). The n^{th} moment, $E\{\beta^n\}$ is the coefficient of $\beta^n/n!$ in the Taylor series expansion of $M_\beta(s)$ and is expressed as

$$M^{(n)}(0) = E\{\beta^n\} \quad (14)$$

where $M^{(n)}(0)$ is the n^{th} derivative of $M_\beta(s)$ evaluated at $s = 0$

It is important to know here that one of the properties of MGF will be made use of in this work. If X has MGF $M_X(S)$, and Y has MGF, $M_Y(S)$, X and Y are RV, x is independent of y, then MGF of $X + Y$ is

$$M_{X+Y}(S) = E\{\beta^{S(x+y)}\} = E\{\beta^{sx}\} * E\{\beta^{sy}\} = M_X(S) * M_Y(S) \quad (15)$$

The implication is that; the combined channel could be referred to as a sum of two fading channel in terms of the expected value definition of MGF or in real MGF term a product fading channel. In this research, the term composite fading channel is adhered to, since it does not matter whether it is a sum or product of fading channels according to (15). The MGFs of the Rayleigh distribution is given by [16] as

$$M_{\beta R}(S) = \frac{1}{1 - S\bar{\beta}} \quad (16)$$

and that of Rician distribution is given by [16] as

$$M_{\beta Ri}(S) = \frac{\alpha}{\alpha - S\bar{\beta}} e^{\left(\frac{KS\bar{\beta}}{\alpha - S\bar{\beta}}\right)} \quad (17)$$

where $\alpha = k + 1$

where k is the Line of Sight (LOS) component of the Rician distribution.

From all the relations given above, it shows that directly or indirectly PDF is a major determinant in finding most of these statistical parameters. These statistical functions have been used in the past to model flat fading channel. In this work, an attempt is made to demonstrate the use of PDF through MGF by first demonstrating how to find the PDF and then used it to find the MGF for the analysis of both BER and P_{out} of WCS channel. It is a theoretical modeling of flat fading WCS channel. It must also be noted that any of these can be used to determine the other.

3.0 Wireless Communication Modelling using PDF

Statistical predictions (modelling) in wireless communication have evolved over a very long time. The theory behind it is interwoven in equations (1) to (12) given previously. PDF can be developed from any of the above equations. This work gives an example of how to use expected values (moments) through the MGF to develop PDF of a combined Rayleigh and Rician distribution.

3.1 Moments of the Composite Fading Channel Model

The n^{th} moment of the composite Rayleigh and Rician fading channel is given by the product of (6) and (7) using one of the properties of MGF as

$$E(\beta_l^n)_c = \frac{\Gamma^2(1+n)}{(k_l+1)^n}\ {}_1F_1(-n; 1; -k_l)\bar{\beta}_l^{2n} \quad (18)$$

This is the n^{th} moment of the composite channel. This research takes a look into an independent and identically distributed (IID) scenario where the instantaneous output SNR per symbol, β, is expressed by Simon and Alouini (2005) as, $\beta_{out} = \frac{E_s}{LN_o}(\sum_{l=1}^{L} R_l)^2$ for EGC, where L is the number of path, when L = 1, it means there is no diversity and L > 1 means that diversity is introduced as either space or antenna diversity.

3.2 Moments of the EGC Output

In EGC with equi-probable transmitted bits, the combined SNR per bits, β_{EGC}, is expressed by Simon and Alouini (2005) as

$$\beta_{EGC} = \frac{E_b\left(\sum_{l=1}^{L} R_l\right)^2}{LN_O} \tag{19}$$

Since the branches are weighted equally then (19) becomes

$$\beta_{EGCout} = \frac{L^b}{\sqrt{L^{1-a+b}}}\bar{\beta} = \frac{L^2}{\sqrt{L^{1-1+2}}}\bar{\beta} = L\bar{\beta} \tag{20}$$

$L\bar{\beta}$ replaces β_l in (18) therefore, (18) for EGC becomes

$$E(\beta_l^n)_c = \frac{\Gamma^2(1+n)}{(k_l+1)^n}\ {}_1F_1(-n;1;-k_l)(L\bar{\beta})^{2n}$$

$$E(\beta_{EGC}^n)_c = \frac{L^{2n}\Gamma^2(1+n)}{(k_l+1)^n}\ {}_1F_1(-n;1;-k_l)\bar{\beta}^{2n}$$

(21)

Where, $\bar{\beta}$ is the average SNR at the output of the combiner.

3.3 MGF of the Output SNR of EGC

The MGF of the output of the l^{th} diversity branch of the combiner could be written as a summation that yields an infinite power series (Taylor series) given as

$$M_\beta(s) = \sum_{n=0}^{\infty} \frac{s^n E(\beta_c^n)}{n!} \tag{22}$$

This implies that, the output SNR of the EGC could be obtained by substituting for $E(\beta_{EGC}^n)_c$ in (22) from (21) will be

$$M_{\beta EGC}(s) = \sum_{n=0}^{\infty} \frac{L^{2n}\Gamma^2(1+n)}{n!(k_l+1)^n}\ {}_1F_1(-n;1;-k_l)(s\bar{\beta}^2)^n \tag{23}$$

$$M_{\beta EGC}(s) = \sum_{n=0}^{\infty} \frac{L^{2n}\Gamma^2(1+n)}{n!(k_l+1)^n}\ {}_1F_1(-n;1;-k_l)Y^n \tag{24}$$

Where, $Y = s\bar{\beta}_l^2$ and $M_\beta(s)$ is for a single channel, and $l = 1,2,\dots .L$ and for L>1 and since this research considered IID fading channel, then, $Y = s\bar{\beta}^2$,

The ultimate goal of this work is to derive the PDF from this MGF. It is quite obvious that when $n = 0$, the coefficient of (24) is 1. The MGFs form an infinite power series in "s" and are not guaranteed to converge for any practical use. The uncertainty in its divergence or convergence could be resolved using Pade Approximation (PA), details of which could be found in [23].[24], [25] and [26].

3.4 PA of the MGF of output SNR

PA is a rational function approximation of an infinite power series which in its natural form may not be useful in further computation. A power series $f(z)$ is expressed in equation (25) where the variable Z is the set of complex numbers,

$$f(z) = \sum_{n=0}^{\infty} D_n Z^n, \tag{25}$$

Where D_n is the set of real numbers

A compact rational approximation of equation (25) is desirable to capture the limiting behaviour of the power series. In order to capture the limiting behavior of the MGF of equation (24), the power series was approximated with PA.

$$M_\beta(s) = \sum_{n=0}^{\infty} \frac{L^{2n}\Gamma^2(1+n)}{n!(k_l+1)^n}\ {}_1F_1(-n;1;-k_l)Y^n = \frac{\sum_{i=0}^{A} a_i y^i}{1+\sum_{j=1}^{B} b_j y^j} = R(y) \tag{26}$$

When L=1, the implication is that there is no diversity. Equations (27) and (28) are the approximated MGF channel model (closed-form expression) for the received signal, in the presence of the composite (Rayleigh and Rician) fading channel at L = 1 and K = 0, 10 are given as

K = 0

$$R(y) = \frac{1-24y+177y^2-444y^3+274y^4}{1-25y+200y^2-600y^3+600y^4-120y^5} \tag{27}$$

K = 10

$$R(y) = \frac{1-6.63y+14.51y^2-11.91y^3+2.87y^4}{1-7.63y+20.97y^2-25.49y^3+13.38y^4-2.34y^5} \tag{28}$$

It should be noted that different results are obtained when L > 1

3.6 PDF of the Composite Rayleigh and Rician Fading Channel

The partial fraction decomposition of the MGF is given below as equation (29) and this is the PDF developed from this partial decomposition

$$P_{\gamma}(y) = f(y) = \sum_{i=1}^{\beta} \lambda_i e^{-P_i y} \tag{29}$$

Using the residues (zeroes) and poles obtained from (26) in (29) and for L = 1, that is, only one antenna, meaning that there is no diversity and K=0 which implies a composite Rayleigh and Racian fading channel. When $K > 0$ then a composite Rayleigh and Rician fading channel is implied. The model equations for EGC technique are derived but the PDF models without diversity are given for L = 1 and K = 0

$$f(y) = 0.004e^{-7.09y} + 0.08\lambda_i e^{-3.6y} + 0.4e^{-1.41y} + 0.52e^{-0.26y} \tag{30}$$

for L = 1, K = 10

$$f(y) = -0.004e^{-3.05y} + 0.06e^{-2.1y} + 0.34e^{-1.35y} + 0.5e^{-0.79y} + 0.11e^{-0.35y} \tag{31}$$

4.0 Application: System Performance Measure

Equations (30) and (31) are consequences of (29) which are useful equations to predict performance of signal transmitted in wireless media. The model equations for BER and P_{out} is given by Simon and Alouini (2005) as

$$P_b = \int_o^{\infty} P_b\left(E/_{\beta}\right) P_{\beta}(\beta)\, d\beta \tag{32}$$

where $P_b\left(E/_{\beta}\right)$ is the conditional error probability and $P_{\beta}(\beta)$ is the PDF of the SNR

$$P_{out} = \int_o^{\beta_{th}} P_{\beta}(\beta)\, d\beta \tag{33}$$

Where $P_{\beta}(\beta)$ is the PDF of β, and β_{th} is the specified threshold

Using (32) and (33) and interpolating it with (27), (28), (30), and (31) considering an M-ary Quadrature Amplitude Modulation, (MQAM), modulated signaling an EGC diversity. Equation (32) for L = 2, K = 0, becomes

$$P_b = \frac{4}{\pi Log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\left[\int_0^{\pi/2} R_{EGC}(y)\, d\emptyset - \left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\pi/4} R_{EGC}(y)\, d\emptyset\right]$$

$$P_b = \frac{4}{\pi Log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\begin{bmatrix} \int_0^{\frac{\pi}{2}} \frac{1-96y+2.83e^3y^2-2.84e^4y^3+7.01e^4y^4}{1-100y+3.2e^3y^2-3.84e^4y^3+1.54e^5y^4-1.23e^5y^5}\, d\emptyset \\ -\left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\pi/4} \frac{1-96y+2.83e^3y^2-2.84e^4y^3+7.01e^4y^4}{1-100y+3.2e^3y^2-3.84e^4y^3+1.54e^5y^4-1.23e^5y^5}\, d\emptyset \end{bmatrix} \tag{34}$$

(32) is the expression obtained for the systems BER performance measures using MQAM modulation technique. P_{out}, in (33) after interpolation is

$$P_{out} = \int_o^{\beta_{th}} f(y)\, dy \tag{35}$$

5.0 Results and Discussion

The PDF is important in statistical modeling of communication channel, showing the statistical distribution that could be used to analyse and predict likely probability of fading and the signal outage in the channel of interest. The PDF model equation in (30) and (31) were used to investigate the BER and P_{out} performance respectively of signal propagating in the composite, product fading channel; Rayleigh and Rician. Using the work of [16], these equations were employed to investigate BER performance as an example on the versatility of the PDF. The model equations derived in this research are shown graphically for EGC at different K = 2, 5, 10 dB in Figures 1and 2. Figure 1 shows the PDF without diversity in Rayleigh and Rician composite fading channel respectively at K = 2, 5 and 10 dB. While Figure 2 depicts the PDF of EGC at L = 2 for K = 2, 5, and 10 dB.

Figure 3 shows the P_{out} at L = 1. In figures 1and 2, the PDF with the highest altitude was the PDF of K = 10 while the lowest was at K = 2. This is not unconnected to the fact that at weak LOS the BER is always high and at strong LOS the BER is expected to be of low value.

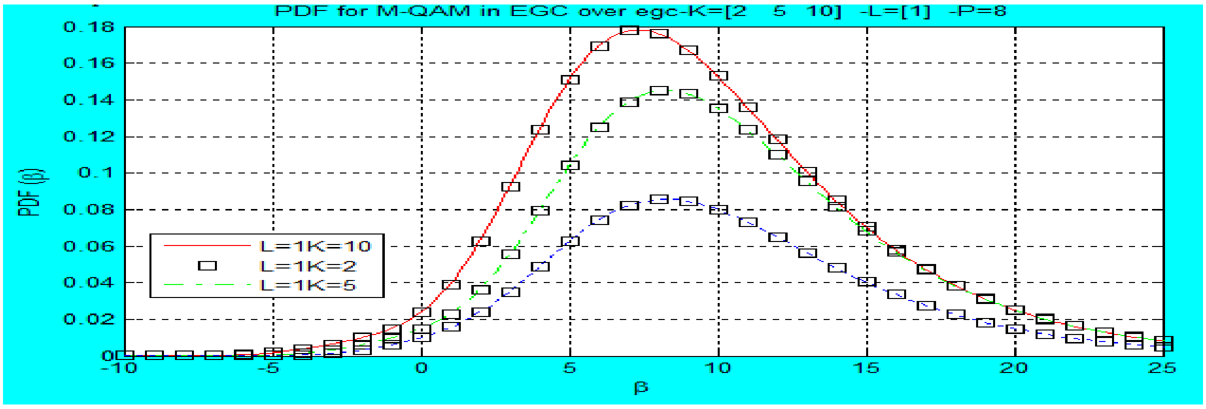

Figure 1: PDF of Rayleigh and Rician Product Fading channel versus SNR with L = 1 (no diversity) at different LOS (K)

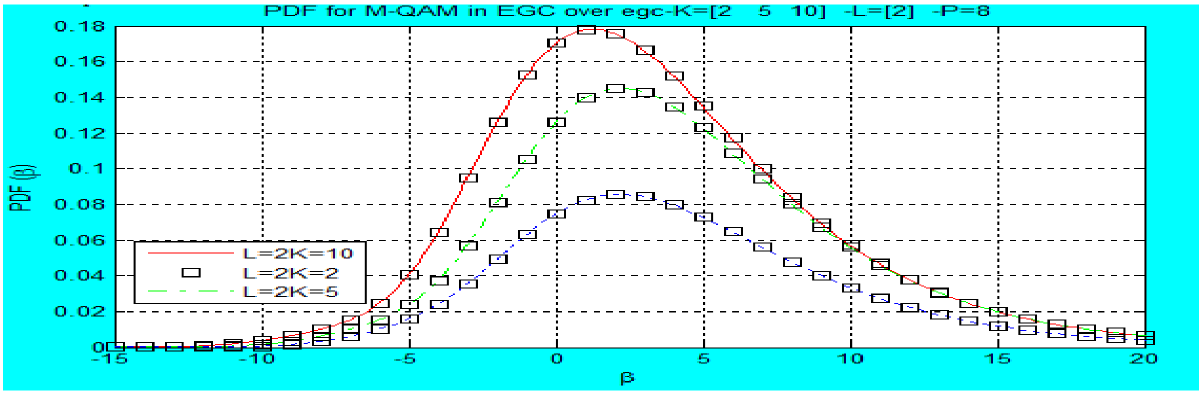

Figure 2: PDF of Rayleigh and Rician Product Fading channel versus SNR with L = 2 over EGC at different LOS (K)

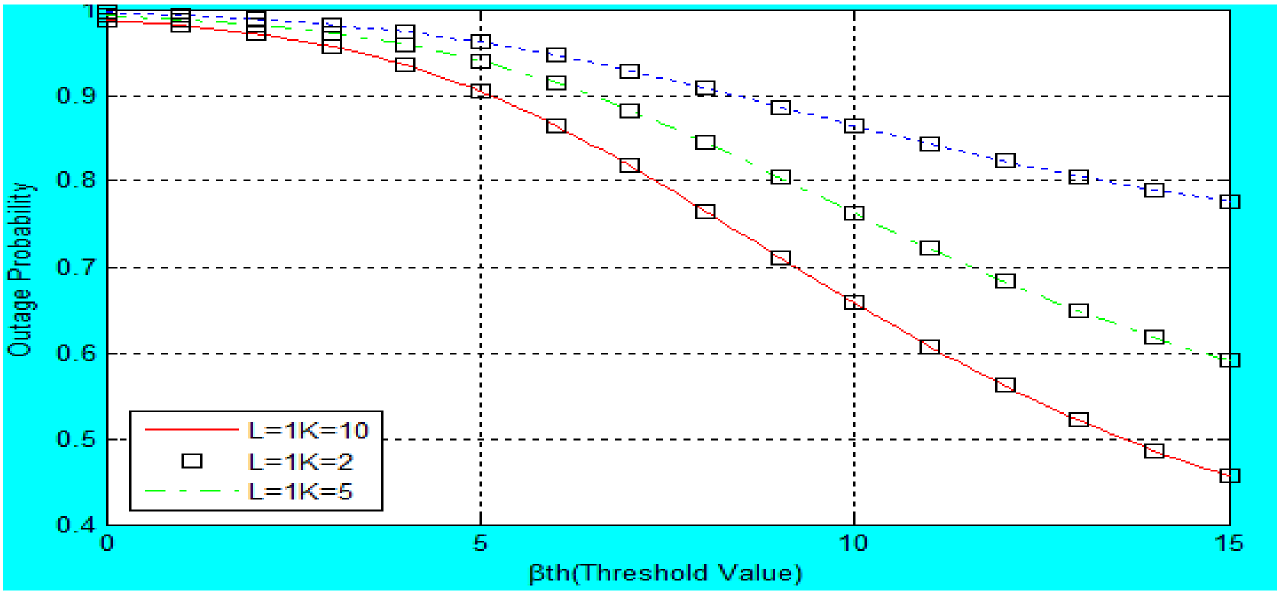

Figure 3: outage probability versus threshold value of Rayleigh and Rician Product Fading channel with L = 1 (no diversity) at different LOS (K = 2, 5, 10)

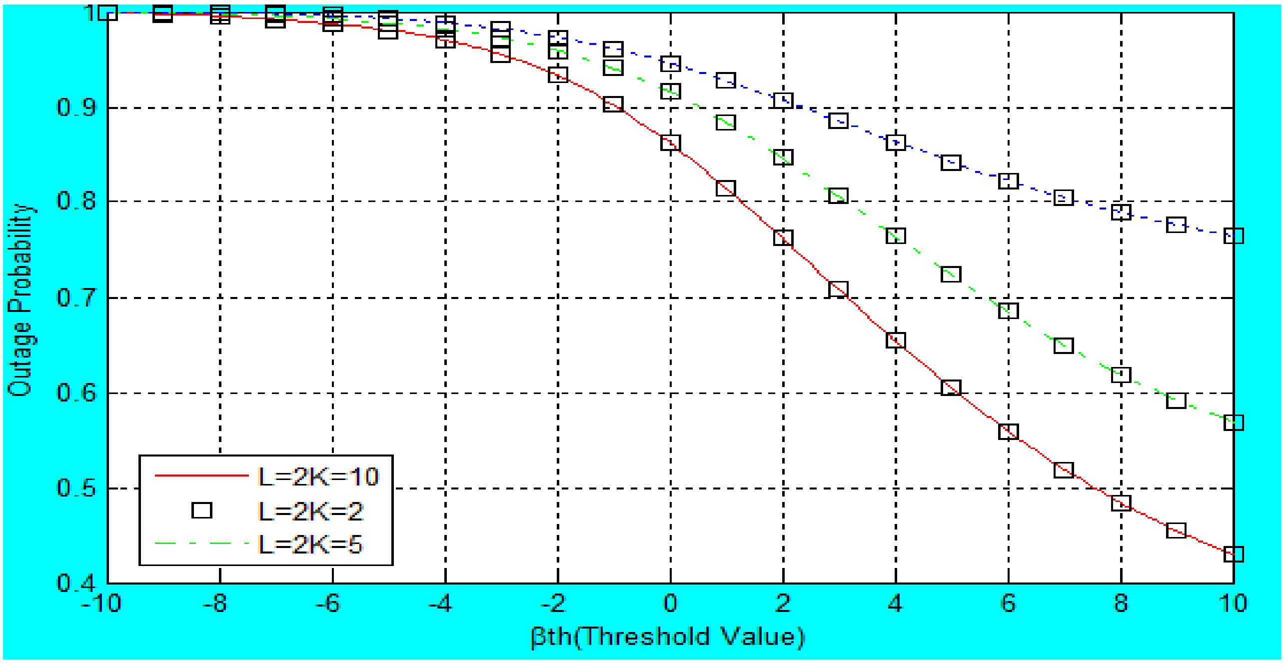

Figure 4: outage probability versus threshold value of Rayleigh and Rician Product Fading channel with L = 2 over EGC at different LOS (K = 2, 5, 10)

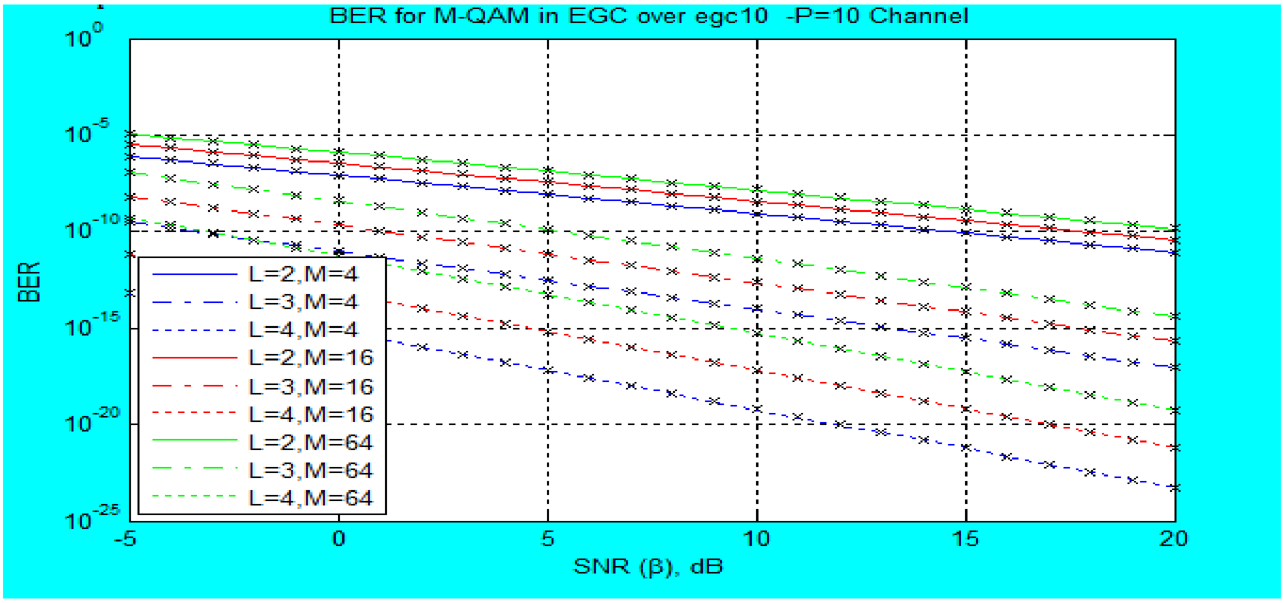

Figure 5: BER of 4-QAM, 16-QAM, and 64-QAM, modulated signal versus SNR over EGC in Rayleigh and Rician Product Fading channel with diversity K = 10

Figures 3 and 4 show the P_{out} for different value of K at L = 1, (when there is no diversity) and L = 2, that is when diversity was introduce. At a threshold value of 5 dB in figure 3 which shows the results without diversity, the P_{out} were 92.19%, 93.96% and 96.17% respectively for K = 10, 5 and 2, while in Figure 4 when diversity was introduced, the P_{out} observed were 60.4%, 72.44% and 87.24% respectively. These results showed that as the threshold value increases theP_{out} reduces, also as the diversity order, number of path increases, the P_{out} becomes very low,but in contrary, as the LOS reduces the P_{out} increases. This is a normal occurrence in practical situation. Figure 5 shows the results for Rayleigh and Rician fading channel at $K = 10$, that is, a verystrong Line of Sight (LOS) components exists. At SNR of 2 dB at $L = 2$, and for 4-QAM and 64-QAM, the BER of 6.59×10^{-8} and 3.32×10^{-6}, respectively, were obtained and at $L = 4$ in a similar scenario BER of 2.12×10^{-16} and 5.8×10^{-12} were also obtained. At SNR of 15 dB, BER obtained for $L = 2$ were 1.64×10^{-10} and 8.47×10^{-9} respectively for 4-QAM and 64-QAM and at $L = 4$ the BER were 5.92×10^{-22} and 5.52×10^{-18} respectively. This shows that, as SNR increases the BER decreases and as the number of paths increases the BER decreases. This means that signal to be transmitted must be of higher SNR for a meaningful reception at the receiver.

6.0 Conclusions

This work has been able to show the use of PDF, which was developed from the expected values through the MGF and terminated by PA in wireless communication. A cascaded Rayleigh-Rician channel employed in modelling satellite communications was used as an example to demonstrate its workability. The PDF versatility was established through BER and outage probability over Equal Gain Combining (EGC) diversity. It has been shown that, in statistical modelling of communication channel, PDF is an important parameter to predict the behaviour of signal propagated in wireless media. These results were validated through simulations hence, the theoretical and simulation results were in agreement.

REFERENCES

[1] Clausius, R.: The Mechanical Theory of Heat, with its Applications to the Steam-Engine and to the Physical Properties of Bodies, *in the Annals of Physics*, *1853; 155:3*

[2] Shannon, C. E.: A Mathematical Theory of Communication, *Bell System Technology Journal 1948; 27:1, 379–423.*

[3] Cover, T. M., and Van Campenhout, M. Maximum Entropy and Conditional Probability *IEEE Transaction on Information Theory, 1981; IT-27: 4, 483-489*.

[4] Mead, L. R., and Papanicolaou, N.: Maximum Entropy in the Problem of Moments, *Journal of Mathematical Physics, 1984 25:8, 2404*

[5] Kale S. S. and Jadhav A.N.: An Empirically Based Path Loss Models for LTE Advanced Network and Modelingof4G Wireless Systems at 2.4 GHz, 2.6 GHz and 3.5 GHz, *International Journal of Application or Innovation in Engineering & Management (IJAIEM), 2013, 9, 252-257*.

[6] Brennan, D.: Linear Diversity Combining Techniques, *Procedure of IRE, 1959, 47: 6, 1075- 1102.*

[7] Ahn, K. S. and Health, R.W.: Performance Analysis of Maximum Ratio Combining with Imperfect Channel Estimation in the Presence of Co-Channel Interferences, *IEEE Transactions on Wireless Communication, 2009, 8 (3):1080-1085*

[8] Aruna, G.: Performance Analysis of MRC Receiver with Channel Estimation error and CCI in Nakagami-m Fading Channels, *IEEE International Conference on Computational Intelligence and Computing Research, 2014; NCC 2012, IIT Kharagpur, India.*

[9] Aruna, G.; PDF methodology of analysis of L branch equal gain combiner with carrier phase error and CCI over Nakagami-m fading, *International Conference on Computing and Network Communications (CoCoNet), Trivandrum 2015; 505-510*.

[10] Saikia and Subadar, R.,: L-MRC Receiver with Estimation Error over Hoyt Fading Channels, *International Journal of Engineering and Technology, 2017;* ***9****(4):2945-2952.*

[11] Papoulis, A.: Probability, Random Variables and Stochastics Processes, 3rd Edition MC Graw -Hill Inc. New York City USA 1991

[12] Sheldon R.: A First Course in Probability, Fifth Edition, Prentice Hall, Inc.Upper Saddle River, New Jersey, USA. 1998

[13] Sklar, B.: Digital Communication: Fundamentals and Applications, Second Edition, Prentice Hall.Upper Saddle River, New Jersey, USA. 2002

[14] Yates, R.D., and Goodman, D.J.: Probability and Stochastic Processes; A Friendly Introduction for Electrical and Computer Engineers, 2nd Edition, John Willey & sons, Inc., III River street, Hoboken, NJ, USA. 2005

[15] Afonja, B., Olubusoye, O. E., Ossai, E., and Arinola, J.: Introductory Statistics: A Learner's Motivated Approach, Revised Edition, Evans Brothers (Nigeria Publishers) Limited, Ibadan, Nigeria, 2014; p. 213

[16] Simon, M. K., and Alouini, M.S.: Digital Communication over fading channels, Second Edition, John Wiley & Sons. Inc. Hoboken, New Jersey, USA. 2005

[17] Gradshteyn I. S. and Ryzhik I. M.: Table of Integrals, Series, and Products Seventh Edition, Academic Press, San Diego, USA.2007

[18] Olver, F. W. J., Lozier, D. W., Biosvert, R. F., and Clark C. W.: NIST Handbook of Mathematical Functions, First Edition,Cambridge University Press, New York, USA. 2010

[19] Peebles, P.Z.: Probability, Random Variables and Random Signal Principles, McGraw-Hill Inc. New York city, USA. 1993

[20] Jabi, M., Szczecinski, L., and Benjillali, M.: Accurate Outage Approximations of MRC Receivers in Arbitrarily Fading Channel, *IEEE Communication Letters, 2012; 16:18, 789-792.*

[21] HelstromC.W.: Probability and Stochastic Processes for Engineers, Second Edition, Macmillan, New York, USA. 1991

[22] Ramanathan V.: Performance Evaluation of Equal Gain Diversity Systems in Fading Channels, M. Sc. Thesis in Electrical Engineering, Virginia Polytechnic Institute and State University USA. 2003

[23] Amindavar H.: Application of Padé Approximation in Signal Analysis, Unpublished PhD. Dissertation, Dept. of Electrical Engineering, University of Washington, Seattle, USA. 1991

[24] Amindavar H., and Ritcey J. A.: Padé Approximation of Probability Density Functions, *IEEE Transactions on Aerospace and Electron System, 1994; 30:2, 416- 424.*

[25] Baker, G. A., and Graves-Morris, P.: Padé Approximants, Third Edition, Cambridge University Press, United Kingdom. 1996

[26] Ismail, M. H. and Matalgah, M. M.: On the Use of Padé Approximation for Performance Evaluation of Maximal Ratio Combining Diversity Over Weibull Fading Channels, *EURASIP Journal on Wireless Communications and Networking,* 2006; 2; 62-68.

[27] Kavehrad M.and Joseph M.: Entropy and Methods of Moments in Evaluation of Probability of Error in Digital Communication, *IEEE Transactions on Communication 1986; 34:12, 1183-1189.*

[28] Stark, H. and Woods, J.: Probability and Random Processes with Applications to Signal Processing, Prentice-Hall, 2006; p.706.