



Effect Numerical Study of a Reactive Nonlinear Mixed Convective Third-Grade Fluid subjected to Newtonian cooling

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Abstract: This study delves into the behavior of a reactive nonlinear mixed convective third – grade fluid under the influence of Newtonian cooling. Utilizing advanced numerical techniques, the research explores the interplay between reactive chemical processes and nonlinear fluid dynamics in a mixed convective region. The governing equation for the third – grade fluid was formulated, incorporating both convective and reactive terms, and solved using a robust computational technique (Spectral Chebyshev Collocation Method), the result was then validated with Shooting Runge - Kutta method. The effects of Newtonian cooling on the fluid's velocity fields were analyzed, with particular attention to how cooling influences the stability and performance of the convective flow. The table and plots revealed the patterns of flow distribution, highlighting the impacts of reactive terms (parameters) and nonlinearities on the overall system behavior. This study provides valuable insights into the velocity management of reactive fluids in industrial applications, offering a deeper understanding of how cooling mechanisms can be optimized to control fluid behavior effectively.

KEYWORDS: Nonlinear Mixed Convection, Third – Grade Fluid, Newtonian Cooling.

1. Introduction

In the realm of fluid dynamics, the study of complex fluid behavior under various conditions provides critical insights into both theoretical and practical applications. One such area of interest is the behavior of third – grade fluids, a class of non – Newtonian fluids characterized by their nonlinear constitutive relations. These fluids exhibit unique viscoelastic properties that significantly influence their flow dynamics, especially when subjected to external forces and reactive processes.

Each management and heat control have been one of the greatest problems encountered by many scientists particularly in the industrial and chemical engineering when a reactive fluid is the tool used. The study investigated the nonlinear mixed convective flow properties of a third – grade flammable fluid in a vertical channel where connections provide wall cooling [1]. The melting effect of the flow of third – grade nonmaterial over a Riga surface was examined. The study also explored the thermophoresis and random diffusion. An irreversibility analysis was conducted using the second law of thermodynamics, which provides insights into the efficiency of energy quality in an isolated closed system [2].

Francesco et al examined the role of hydrogen in the current global energy transition process. Hydrogen is gaining traction as a potential fuel to reduce carbon – dioxide emissions. Numerical

simulation of hydrogen combustion necessitates an additional consideration, as many existing models designed for hydrocarbons may not be suitable [3].

Patrick Rorimpandey et al emphasized that the dual – fuel combustion approach, which entails injecting a small amount of diesel as an ignition source before, during or shortly after the main fuel injection, has garnered significant interest within the engine community. The study revealed that previous studies integrating ammonia into existing natural-gas marine engines have shown a reduction in carbon emissions while maintaining adequate combustion performance [4]. Brindley et al investigated the criteria for combustible fluids. It was noted that many liquids prone to combustion issues in hot lagging are hydrocarbon-based such as heat transfer fluids and diesel fuel. They were studied as representative volatile hydrocarbons within the temperature range of 450 - 500k (177 - 227 °C) [5].

Pablo Fernandez- Yanez et al examined the computational method to study heat transfer in a helicopter. The study revealed that fuel consumption and emissions can be decreased by putting waste heat recovery systems into place and this will help the aviation sector become more friendly and efficient [6].

Fabio Berni et al reported that heat transfer was found to be advantageous in the conversion of a marine Diesel engine. The study confirmed that combination of ammonia and hydrogen demonstrated improved performance compared to using either fuel alone. It also found that $NH_3 - H_2$ mixture could serve as a viable alternative to conventional fossil fuel, provided that issues related to autonomy and no emissions are addressed and their production is based on renewable sources [7].

Xinwen Chen et al performed a quantitative analysis on the dynamics and heat transfer characteristics of droplet on various vibrating surfaces. The study revealed that as vibration frequency increases, surface fluctuations on the droplet intensify, which enhances droplet convection and enables the droplet to enter the nucleate boiling phase more quickly [8]. Zeng et al in experimental investigation of convective heat transfer in the aircraft cabin environment at low air pressure was considered, distributions were examined in a real aircraft cockpit under both low pressure and normal atmospheric pressure [9]. Adesanya et al examined the entropy production rate in the nonlinear convective flow of a reactive pair stress liquid through a saturated material-filled channel that is exposed to convective cooling. Analytical solution for the linked nonlinear boundary-value problems resulting from the mathematical formulation were obtained using the Homotropy Analysis Method (HAM) [10].

Liu et al investigated numerical study of the laminar natural convection heat transfer from two horizontally attached horizontal cylinders where a new model proposed by Bejan et al (1995) was applied in the study and it has been proved to be a more accurate and effective method for numerical simulation of the natural convection in free space than other previous models used previously [11].

The Numerical study of a reactive, non- Newtonian third – grade fluid under mixed convection and Newtonian cooling applies directly to the thermal management and processing of thick combustible polymers, heavy industrial lubricants, and exothermic chemical reactors.

Polymer Manufacturing: Assists in predicting behaviour and preventing degradation in heavy molecular weight polymer melts that exhibit non – Newtonian and heat – generating characteristics thermal Safety and Runaway Prevention.

Thermal Safety and Runaway Prevention: Models how convective wall cooling (Newtonian cooling) helps avert dangerous “thermal runaway” and sudden temperature spikes in chemically reactive fluid flows.

Advanced Heat Exchangers: Optimizes cooling and heating efficiency in devices handling viscous fluids by factoring in temperature – dependent variable viscosity and thermal conductivity.

Microchannel and Porous Media Systems: Predicts velocity profiles and pressure drops where porous matrices restrict fluid motion and exothermic reactions release internal heat.

Nomenclature

(x', y')	Dimensional coordinates (m)	h_1, h_2	heat transfer coefficients
T	Fluid temperature (K)	R	Universal gas constant J / (K.mol)
P	Pressure (Nm ⁻²)	J, J_o	Applied electric current (Am ⁻²)
K	Porous media permeability	M_o	Magnetization constant
C_o	Initial concentration species (mol)	B_i	Biot number
m	Reaction exponent	u	Fluid dimensionless velocity
E	Activation energy (E / mols)		
B	Magnetic induction (tesla)		Greek symbols
l	Magnets – electrodes width	μ	Fluid dynamic viscosity (Ns / m ²)
H	Modified Hartmann number	ρ	Fluid density (kg / m ³)
C_f	Forchemier factor	ν	Vibration frequency (Ns / m ²)
α_1, β_3	Third – grade material parameters	σ	Electrical conductivity (Sm ⁻¹)
(x, y)	non – dimensional Cartesian coordinates	α	Viscosity variation parameter
A	Reaction rate constant	θ	dimensionless temperature
k	Thermal conductivity (Wm ⁻¹ K ⁻¹)	l	Planck 's number (Js)
Q	Heat of the reaction (J)		

It was noted that natural convection heat transfer occurs in situations without forced velocity, relying instead on buoyancy forces arising from the temperature-dependent density gradients within the fluid. Azadeh Jafari et al employed the Chebyshev points based on a spectral collation method to discretize the resolvent operator in the wall normal direction which was formed sufficient from a grid resolution study, as for $N > 400$ the singular values were found to converge to within $0 (10^{-4})$ [12]. Adeosun and Ukaegbu (2022) defined spectral collocation method as an approximate solution method for differential equation of the form; $L(U(Z)) + g = 0$ with boundary condition (Bcs) in the domain D. L denotes differential operator on dependent function U(Z) and

g is known function. A solution satisfying boundary condition (Bcs) is assumed for $U(Z)$ as

$$u = \sum_{i=0}^N d_i T_i(2z-1)$$

Here, d_i are the unknown coefficients of trial functions $T_i(2z-1)$ to be determined. $T_i(2z-1)$ are shifted Chebyshev base function from $[-1,1]$ to $[0,1]$. In the method used, interval of the domain determines the solution. If the domain is large, we have exponential function [13].

Thambair Nair et al considered a collocation method for solving Fredholm integral equations of the first kind which is known to be an ill- pose problem. It was emphasized that an unregularized application of this method can yield reliable results when the rates at which smallest singular values of the collocation points serves as regularization parameter [14].

Their critical applications of thermal analysis are not limited to [15 – 25] and references cited therein under different few assumptions.

Therefore, the primary aim of this study is to investigate the effects of nonlinear parameter on third – grade fluid subjected to Newtonian cooling.

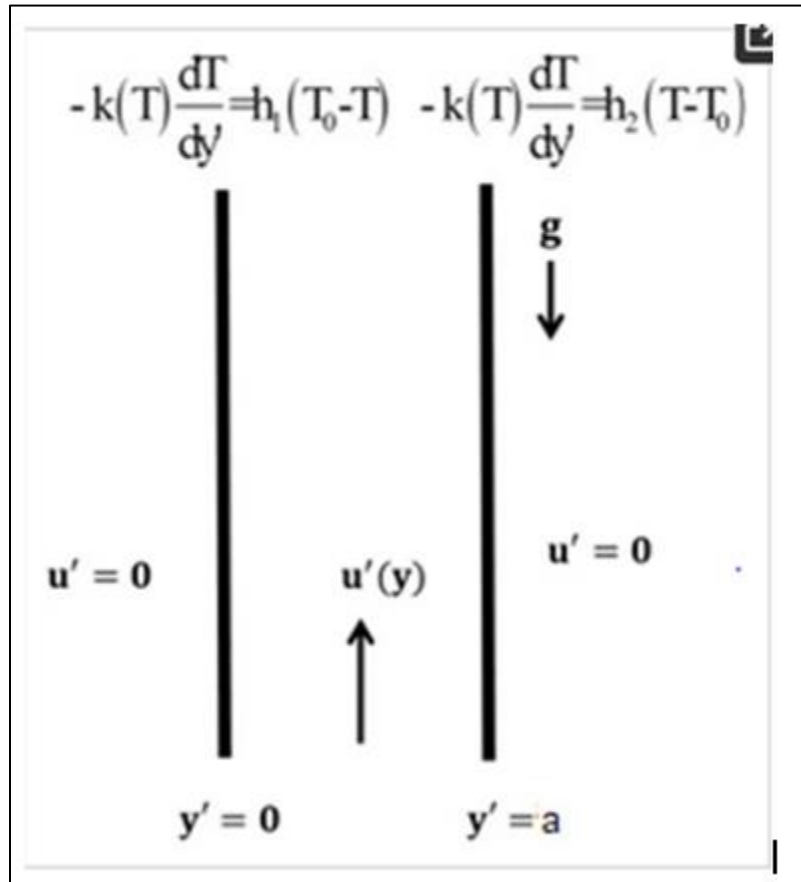


Figure 1: Schematic diagram of the problem.

Problem formulation:

This study aims to investigate the numerical study of a reactive nonlinear mixed convective third – grade fluid subjected to Newtonian cooling as well as the effect of nonlinear convection on it. Behaviour of a reactive nonlinear mixed convective third – grade fluid was carefully studied. Governing equation for the third – grade fluid was thereafter formulated.

The governing equation on nonlinear incompressible fluid is:

Momentum Equation:

$$0 = -\frac{dP'}{dx'} + \frac{d}{dy'} \left(\mu(T) \frac{du'}{dy'} + 2\beta_3 \left(\frac{du'}{dy'} \right)^3 \right) - \left(\mu(T) + 2\beta_3 \left(\frac{du'}{dy'} \right)^2 \right) \frac{u'}{k} - \frac{Cf \rho u'}{\sqrt{k}} + \frac{\pi}{8} J_0 M_0 \ell^{-\frac{\pi}{l} y'} + \rho g \beta (T - T_0) \text{-----} (1)$$

Together with the no slip and nonmoving wall conditions

$$\begin{aligned} u' &= 0, & y' &= 0; \\ u' &= 0, & y' &= a; \end{aligned} \quad (2a)$$

The dependence of viscosity is assumed to be of the form:

$$\mu(T) = \mu_0 \ell^{-\bar{\alpha}_1(T-T_0)} \approx \mu_0 \left(1 - \bar{\alpha}_1 (T - T_0) \right), \bar{\alpha}_1 \leq 1 \quad (2b)$$

Where α_1 is a viscosity variation parameter and μ_0 is the fluid dynamic viscosity.

The momentum equation above was derived using the dimensionless parameters.

$$(x, y) = \frac{1}{a} (x, y), u = \frac{u' \rho a}{\mu_0}, \theta = \frac{(T - T_0)}{RT_0^2}, \theta_a = \frac{(T_a - T_0)}{RT_0}, \partial = -\frac{\pi}{l} a \text{.....} (3)$$

$$0 = -\frac{dP'}{dx'} + \frac{d}{dy'} \left(\mu(T) \frac{du'}{dy'} + 2\beta_3 \left(\frac{du'}{dy'} \right)^3 \right) - \left(\mu(T) + 2\beta_3 \left(\frac{du'}{dy'} \right)^2 \right) \frac{u'}{k} - \frac{Cf \rho u'^2}{\sqrt{k}} + \frac{\pi}{8} J_0 M_0 \ell^{-\frac{\pi}{l} y'} + \rho g \beta_0 (T - T_0) + \rho g \beta_1 (T - T_0)^2 \text{.....} (4)$$

The boundary conditions are as follows:

$$u = 0, \frac{d\theta}{dy} = Bi_1 (\theta - \theta_a) \quad y = 0 \quad (5)$$

$$u = 0, \frac{d\theta}{dy} = Bi_2 (\theta_a - \theta) \quad y = 1 \quad (6)$$

$$S_f = \frac{\rho a^2 C_f}{\mu_0^2} = (1 - a\theta) \frac{du}{dy} + 2\gamma \left(\frac{du}{dy} \right)^3 \quad / \quad y = 0 \quad Nu = \frac{a \in q_w}{KRT_0^2} = \frac{d\theta}{dy} \quad / \quad y = 0 \quad \text{-----} (7)$$

Method of solution

The dimensionless equations (5) – (7) are solved by applying the spectral Chebyshev collocation technique, in this regard; we assume an approximate solution $\phi_j(y)$ as a sum of $N + 1$ basis functions where

$$u(y) \approx U^N(y) = \sum_{j=0}^N b_j \phi_j(y),$$

$$\theta(y) \approx \theta^N(y) = \sum_{j=0}^N C_j \phi_j(y) \quad \text{-----} (8)$$

In (8), $\phi_j(y)$ represents the infinite or spectral Chebyshev polynomials with the undetermined coefficients $(b_j C_f)$

Substituting (8) in the coupled problems leads to the following residues that must be evaluated at specific collocation points.

It is evident that y_1 are points within $[0, 1] = [y_0, y_N]$. Then the Gauss – Lobato points are

$$y_j = \frac{1}{2} \left(1 - \cos \left(\frac{j\pi}{N} \right) \right), \quad j = 0, 1, 2, \dots, N$$

The residues are evaluated at the points

$$R_1(y_j) = 0, \text{ and } R_2(y_j) = 0, 1, 2, \dots, N$$

Their derivatives for the dependent variables are obtained as

$$\frac{d'u}{dy'} = \sum_{j=0}^N b_j \frac{d'u_j}{dy'}, \text{ and } \frac{d^r\theta}{dy^r} = \sum_{j=0}^{N_p} C_j \frac{d^r\theta_j}{dy^r}$$

The differentiation matrices at each Gauss – Lobato points are

$$\bar{u} = (u(y_0), u(y_1), \dots, u(y_N))^T$$

$$\bar{\theta} =$$

These are used to convert the coupled nonlinear boundary – value problem into a set of algebraic equations. For the validation of the spectral collocation weighted residual method, the equations are solved separately by implementing the shooting – Runge – Kutta method on the ND solve algorithm in Wolfram Mathematica Symbolic package.

The table below shows the validation for numerical results for velocity when

$$m = 0.5 = k, H = 1 = R = Gr = G = \partial = 15 = \gamma, \lambda = 0.6, \varepsilon = 0.2 = \alpha, Bi_1 = 20 = Bi_2, N = 35$$

y	U(y)Rk4	U(y)CWRM	ERROR
0	0.	$-6.961955492590054 \times 10^{-18}$	$3.496127692903528 \times 10^{-10}$
0.1	0.04313186505230489	0.04313941885340904	$4.050838935121259 \times 10^{-9}$
0.2	0.07812152389958144	0.07813634077297339	$4.341068185476082 \times 10^{-9}$
0.3	0.10404877297531484	0.10406977160813229	$4.092762989627019 \times 10^{-9}$
0.4	0.12004468954075675	0.12006990360382683	$3.902684536649659 \times 10^{-9}$
0.5	0.12545854595662603	0.12548527366302917	$3.537199116943057 \times 10^{-9}$
0.6	0.12004466918428837	0.12006990360382683	$3.055771372051374 \times 10^{-9}$
0.7	0.10404875393946095	0.1040697716081323	$2.727355438714163 \times 10^{-9}$
0.8	0.07812151422810008	0.07813634077297338	$2.541312688064678 \times 10^{-9}$
0.9	0.04313186461189424	0.043139418853409044	$2.271236268502896 \times 10^{-9}$
1.0	$-9.815129616965232 \times 10^{-10}$	$-7.691962855888261 \times 10^{-18}$	$2.003265362621187 \times 10^{-9}$

Results and discussion:

This part delves into a more specialized investigation: The numerical evaluation of heat irreversibility in the context of porous medium combustion involving third – grade fluids under the influence of Newtonian cooling.

Heat irreversibility, a critical aspect of thermodynamic analysis, represents the inefficiencies and entropy generation within a system. In the realm of porous media combustion, this irreversibility can significantly impact performance and efficiency. The third – grade fluid model, which accounts for non – Newtonian behavior with additional rheological effects, adds complexity to the traditional analysis of heat transfer and fluid flow.

The interaction between third – grade fluids and porous media introduces unique challenges. These fluids exhibit complex flow behavior that can affect the combustion process, heat transfer rates, and the resultant irreversibility. Furthermore, when subject to Newtonian cooling – a process wherein the cooling effect is modeled as proportional to the temperature difference between the fluid and its surroundings – the system’s performance and efficiency are further influenced. We began with a detailed formulation of the governing equations for third – grade fluids in porous media, followed by the implementation of these equations in a numerical framework. The focus will be on quantifying heat irreversibility and analyzing how different parameters influence this irreversibility.

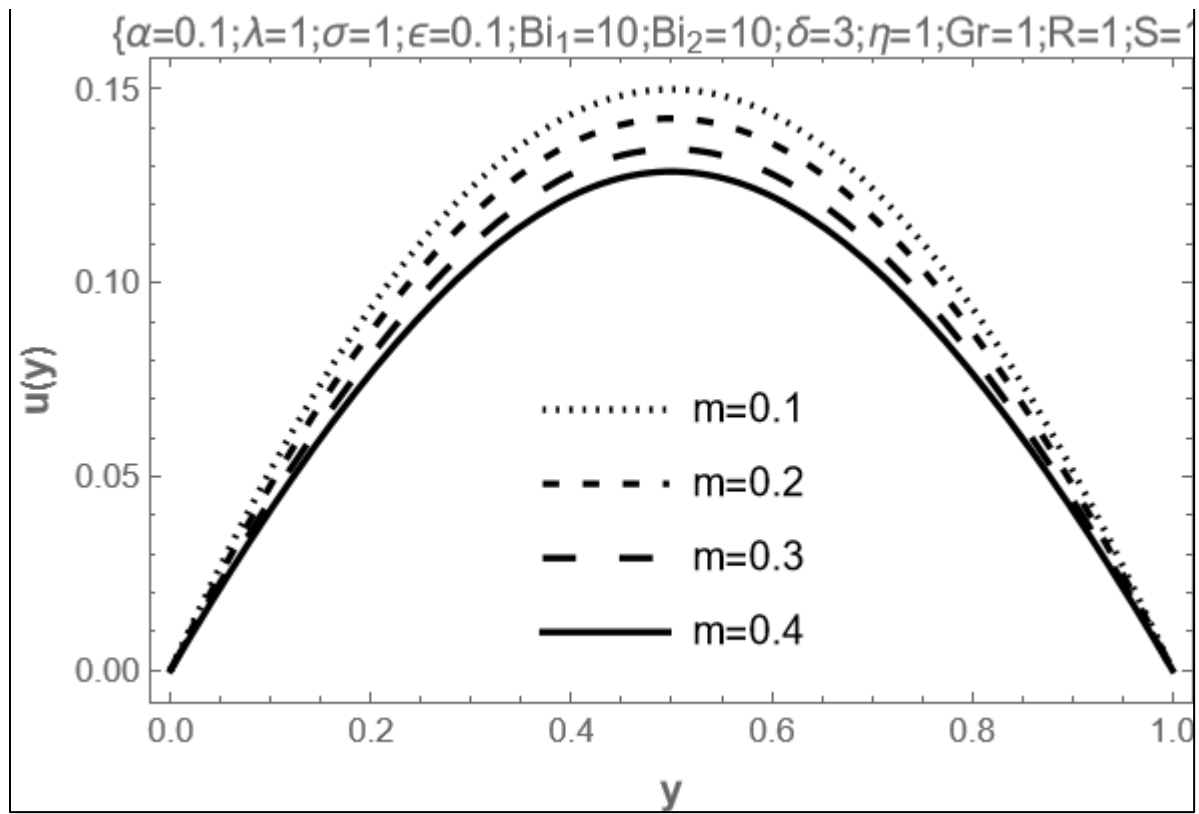
Through numerical simulations and sensitivity analysis, we will explore the interplay between fluid rheology, porous media characteristics and cooling effects.

The outcomes of this aspect are expected to offer valuable insights into the optimization of combustion processes and cooling strategies in engineering applications involving complex fluids. By understanding and mitigating heat irreversibility, we can enhance the efficiency and performance of systems reliant on porous media combustion.

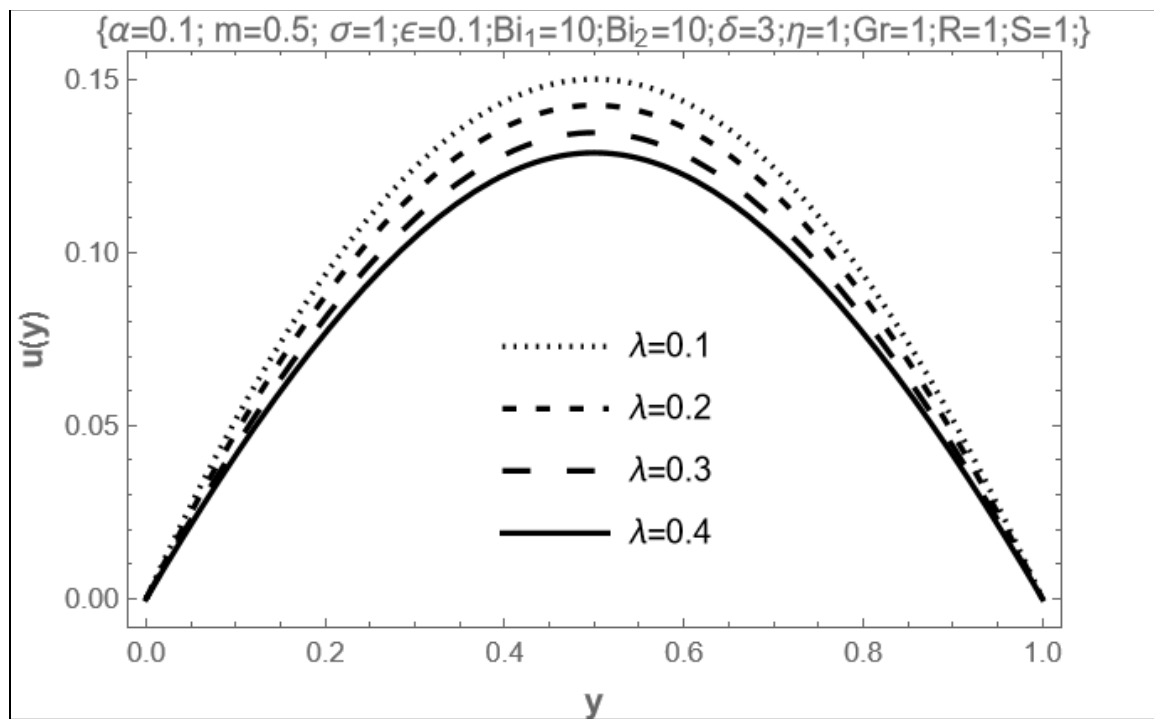
The graphical and tabular results of the special case are presented here.

Existence of a convergent solution to a reactive nonlinear mixed convective third – grade fluid subjected to Newtonian cooling.

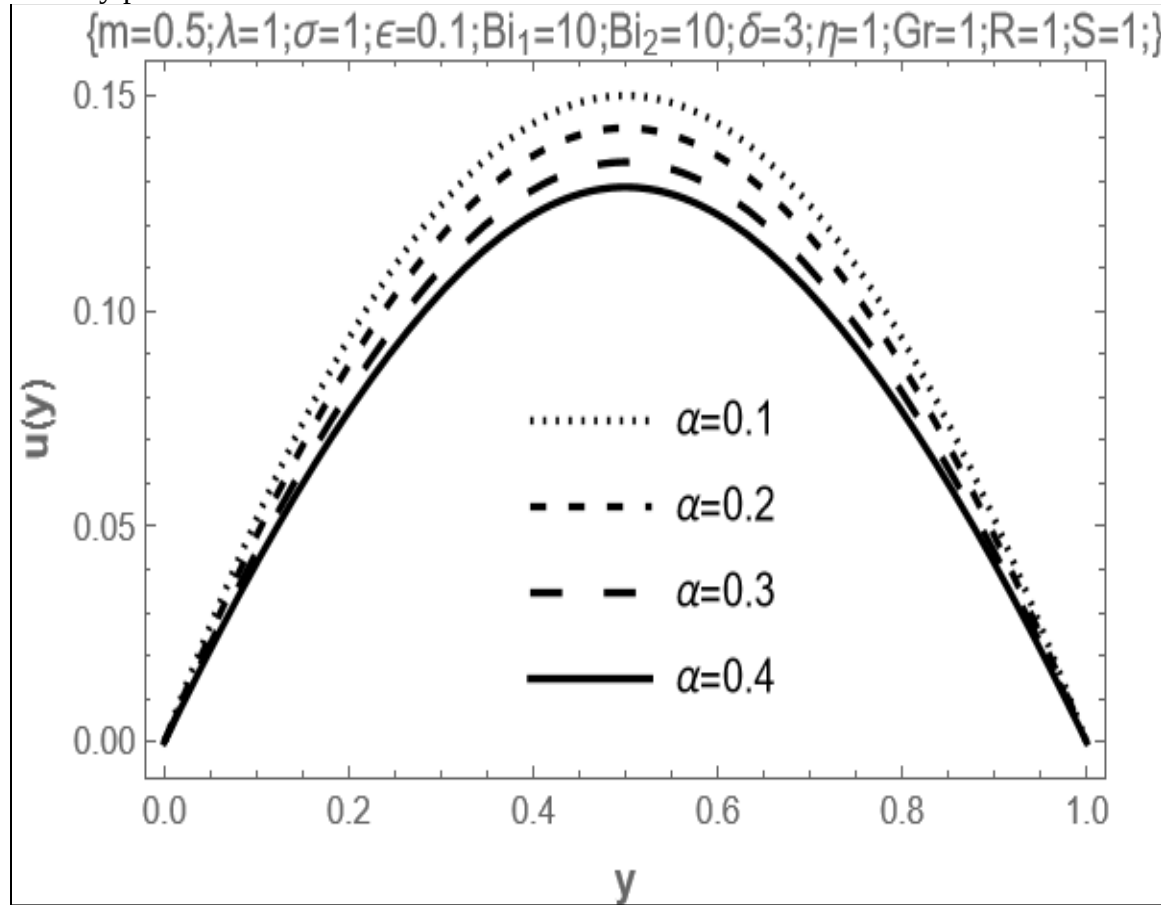
In the computation of unique solutions, the results of the spectral Chebyshev collocation method. Reframing the results, shows complete alignment between the two solutions.



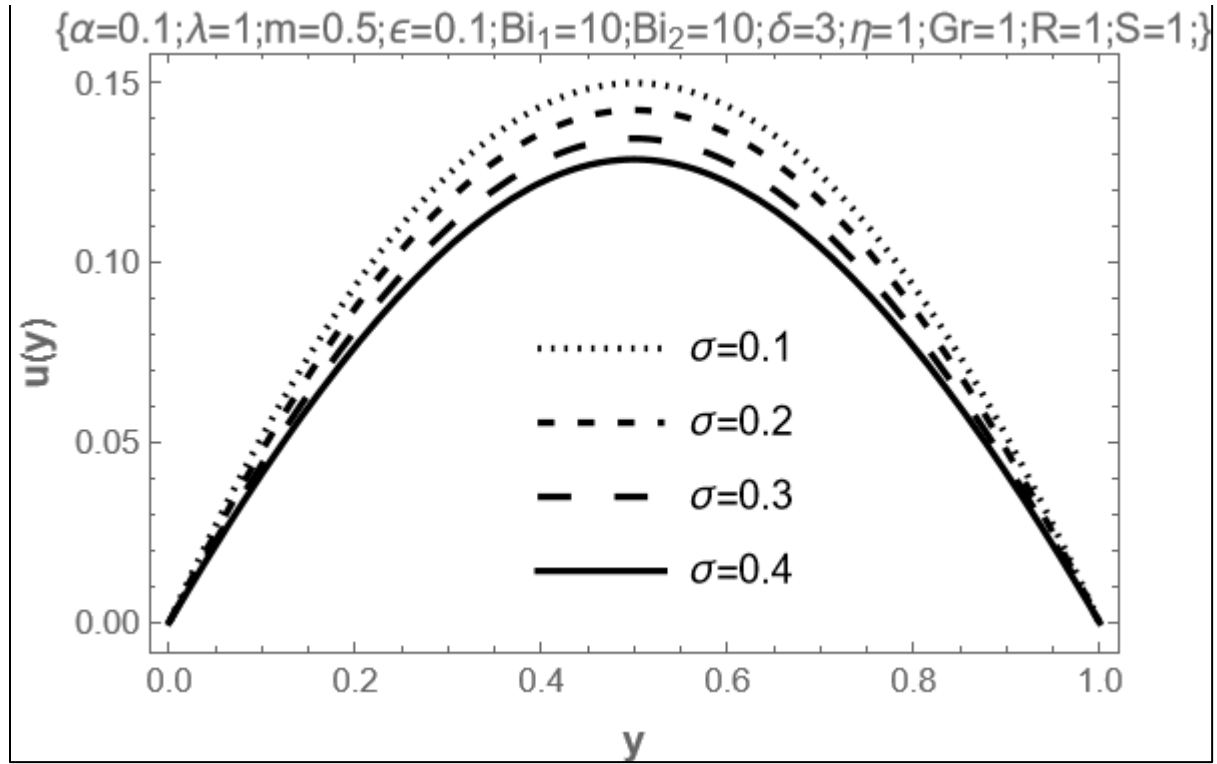
Effect of Reaction Exponent on Velocity Profile: Decrease in the reaction exponent results in increases in the velocity.



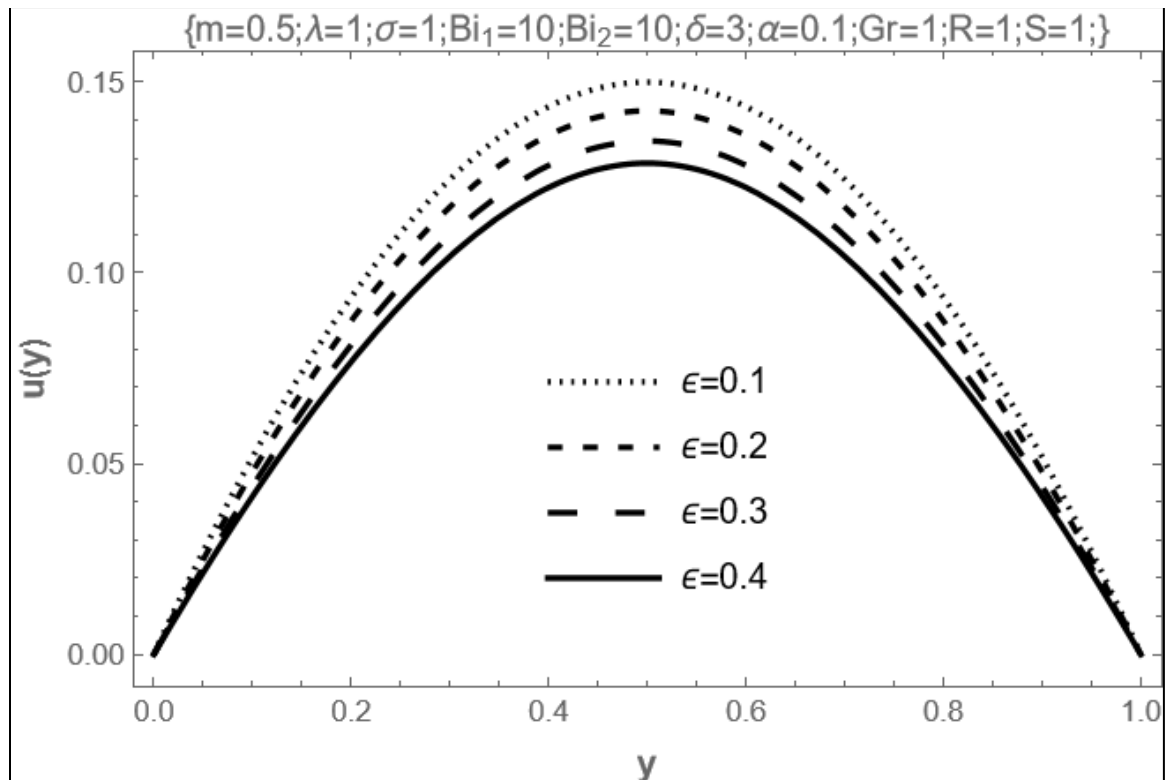
Effect of λ (Lambda) on Velocity Profile: An increase in Lambda (λ) leads to an increase in the velocity profile.



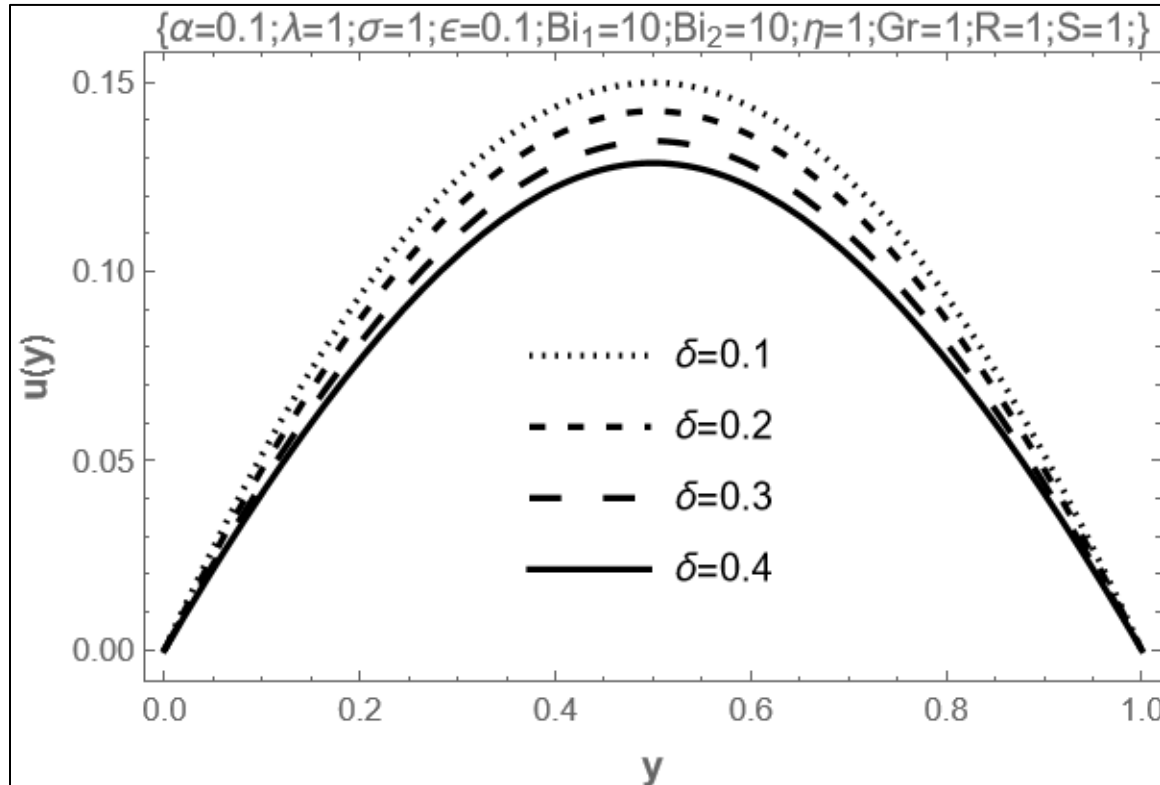
Effect of Viscosity variation parameter (α) on velocity profile: Viscosity has significant effect on the velocity profile of fluid flow. Lower viscosity tends to result in more pronounced, parabolic profiles in laminar flow, while higher viscosity results in flatter, more uniform profiles, especially in turbulent flow.



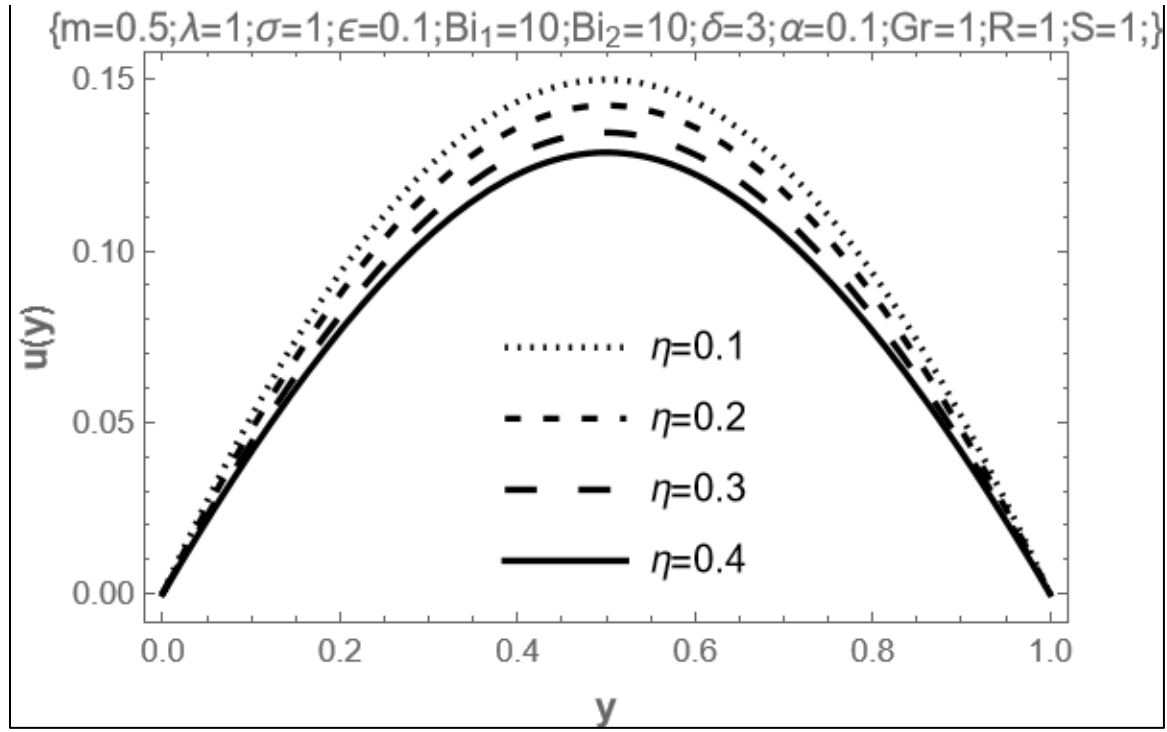
Effect of Electrical conductivity (σ) on velocity profile: An increase in the electrical conductivity leads a decrease on the velocity. The maximum value of the velocity is 0.15 at a point when the electrical conductivity has the least value. This showed that electrical conductivity does not enhance the velocity of the fluid.



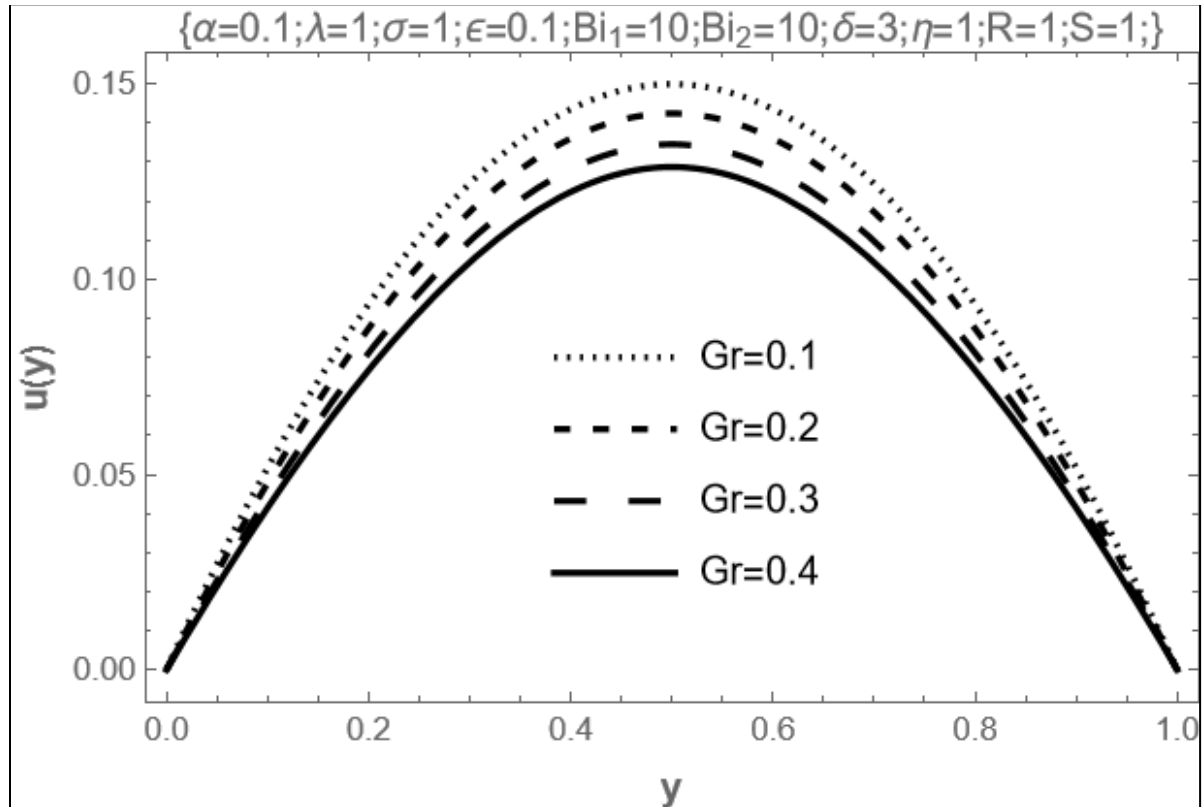
Effect of activation energy (ϵ) on velocity profile: The graph shows that when the activation energy increases, there is a decline on the velocity profile. Therefore, an increase in this parameter does not support the flow of the fluid. In a reactor, higher activation energy means a slower reaction rate at a given temperature. In a catalytic reactor, more efficient catalysis can lead to how reactants and products flow through the reactor. Higher activation energy might results in slow reaction. Activation energy can alter fluid viscosity and influence overall flow dynamics.



Effect of Delta (δ) on velocity profile: Delta (δ) often represents the boundary layer thickness. As delta increases, the velocity profile within the boundary layer changes. For a larger boundary layer thickness. The velocity gradient near the wall becomes less steep, the flow velocity increases gradually from the wall to the free stream and the overall velocity profile becomes more blunt near the wall.

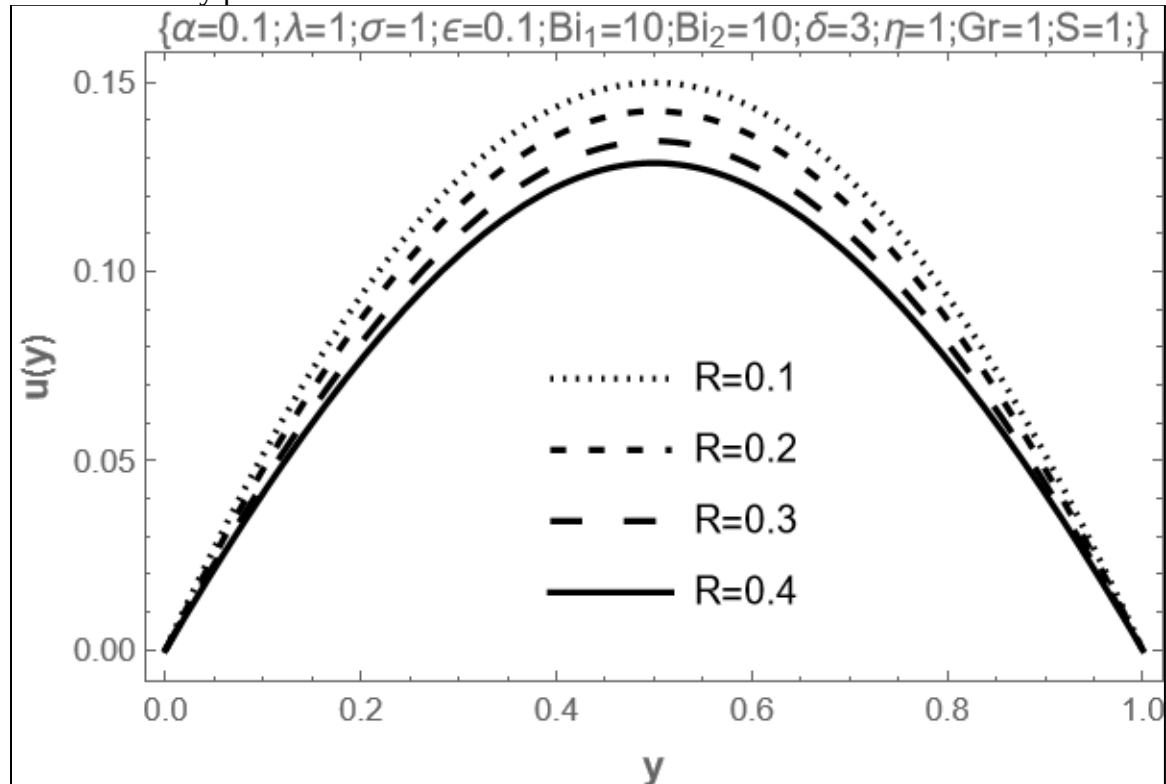


Effect of $\tau(\eta)$ on velocity profile: The velocity of the fluid increases with a decline on the parameter. This shows that a decrease in the parameter enhances the flow of the fluid.

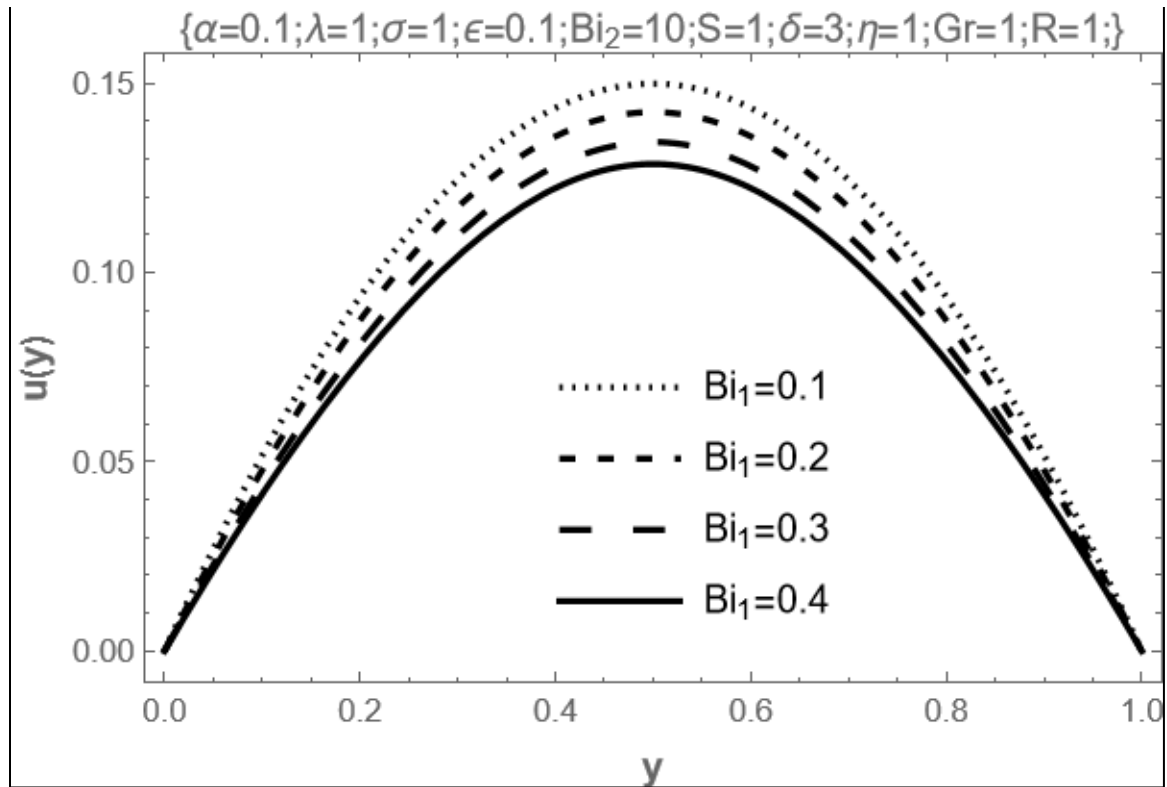


Effect of Grashof number (Gr) on velocity profile: Grashof number (Gr) is a dimensionless number

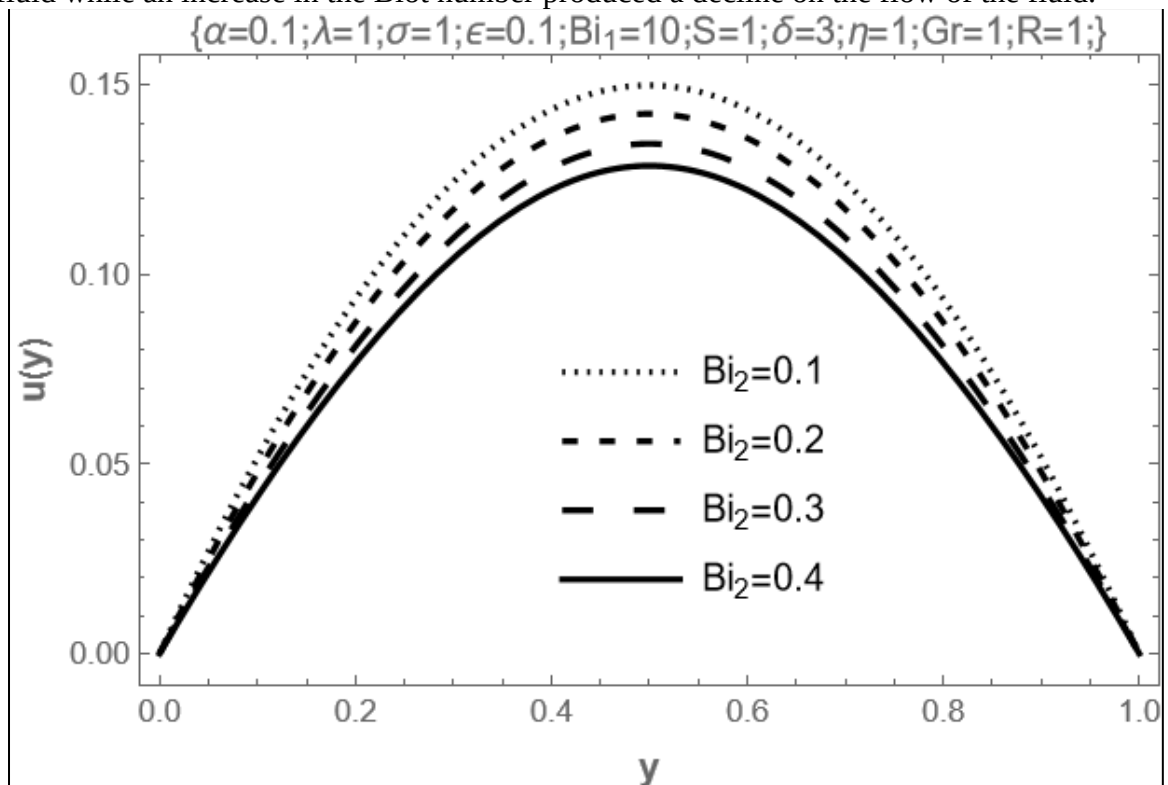
used in fluid dynamics and heat transfer to describe the relative significance of buoyancy forces compared to viscous forces in a fluid. It is clear from the graph that an increase in the Grashof number does not enhance the flow of the fluid. Low Grashof number in laminar flow produces weak buoyancy forces compared to viscous forces. The velocity profile is typically parabolic with low Grashof number. Intermediate Grashof number produces turbulent flow with a complex and chaotic velocity profile.



Effect of Universal gas constant (R) on velocity profile: Higher values of the value of the universal gas constant resulted into decline on the velocity profile as shown in the graph above.



Effect of Biot number on velocity Profile: Low biot number resulted into an increase in the flow of the fluid while an increase in the Biot number produced a decline on the flow of the fluid.



Effect of Biot number on velocity profile: The graph showed that as the Biot number increases, the flow of the fluid decreases. This implies that a decrease in the Biot number enhanced the system.

Conclusion:

This study offers a comprehensive mathematical and numerical investigation aimed at developing an effective model for heat transfer.

It focuses on reactive nonlinear mixed convection of a third – grade fluid subjected to Newtonian cooling, emphasizing key concepts such as combustible fluids, heat transfer, Riga plates, and spectral collocation method.

The analysis includes nonlinear convection with the goal of providing robust solutions for fluid velocity through the numerical resolution of the non – dimensionalized solutions for fluid velocity through the boundary value problem. The accuracy of computations is validated by comparing results obtained via Chebyshev Weighted Residual Method (CWRM) with those from an alternative numerical approach, specifically the shooting RK – 4 method. The validity of the findings is further confirmed through comparisons with results from Runge – Kutta shooting quadrature and the Chebyshev Weighted Residual Methods (CWRM).

Graphical representations illustrate how various parameters including reaction exponent, Lambda, viscosity variation, electrical conductivity, activation energy, Delta, Tau, Grassof number, universal gas constant and Biot number affect velocity profiles. These simulations have direct relevance to innovative techniques in thermodynamics, where velocity effects are induced in the third – grade fluid combustion process. The graphical representations highlight the impact of various physical parameters on fluid velocity.

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