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RADIATIVE HEAT TRANSFER OF NON-NEWTONIAN BLOOD FLOW THROUGH A NARROW ARTERY IN THE PRESENCE OF MAGNETIC FIELD

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Abstract: The problem of non-Newtonian blood flow through a narrow artery is theoretically investigated where the non-Newtonian rheology of the flowing blood is characterized by the Casson fluid model. Thermal radiation, pressure gradient, slip condition, variable viscosity and viscous dissipation are considered within the fluid domain. The governing equations are analyzed using similarity transformation and solved numerically. The effects of major parameters on blood flow and thermal distribution is graphically represented and discussed. The investigation showed that thermal radiation absorption is suitable for increasing blood temperature which could be ideal in some pathological circumstances that need it with negligible danger to other body cells and tissues.

Keywords: **Blood flow, Magnetic effect, radiation, pressure gradient, velocity slip.**

1. Introduction

Radiation as a form of heat transfer, is an important part of life. Taking the instance of medicinal situations, radiation is often used to sterilize equipment, spot broken bones and other organ anomalies, kill cancerous tissues, reduce the size of tumour and in the therapeautic procedures of hyperthermia. Also in rehabilitation purposes, heat therapy is often been use for increasing the extensiblity of collagen tissues, reducing pain, reducing inflammation, edema, relieving muscle spasms, making joints less stiff, and also increases blood flow rate(Zee, 2002; Misra and Sinha, 2013; Ngufor, 2016). In view of these tremendous benefits of radiation, numerous research works both theoretical and practical, have been conducted on the subject. But not too many theoretical analysis on radiative heat transfer involving blood exists in the literature. However, Ogulu and Bestman (1994) conducted a theoretical exploration of blood flow with radiative heat transfer. Later, He *et al.* (2006) discussed the effect of temperature on blood flow in human breast tumor underlaser radiation. Prakash and Makinde (2011) examined the radiative heat transfer on blood flow through a stenotic artery in the presence of magnetic field. They reported that both the blood temperature and velocity increases with radiation absorption.Blood exhibits some magnetohydrodynamic tracts in view of the presence of erythrocytes (a

negatively charged constituent of blood). This ability of blood to respond to magnetic strength provides a basis for research in this wise. In curing some localized diseases, getting drugs or medications to the intended tissue has posed a big problem as every vessel in the circulatory system is linked. Consequently, such medications spread to both intended and non-intended body parts damaging the healthy tissues. Magnetic drug targeting which is the best way out, is still been explored to ensure a near 100% success in curing such diseases. The possibility of using magnetic based principles in the rational treatment of arterial hypertension was investigated by Vardanyan (1973). Founded on their study on the effect of magnetic field on blood flow in the artery, Misra and Shit (2007) concluded that based on the magnetic field's tendency to reduce blood flow through arteries, wall shear stress and the volumetric flow rate, applying external magnetic field in the clinical treatment of hemodynamic diseases can be very useful.

According to the investigation reported by Sinha et al. (2016), the blood velocity and the velocity gradient at the vessel wall gets diminished by the application of sufficiently strong magnetic field. They further reported that their results will be very useful to surgeons who require that the blood flow level is kept at a certain level in the cause of a surgical procedure. Mamata (2017) studied the MHD flow of blood with heat transfer in an arterial segment under the effect of periodic body acceleration. They reported that the axial velocity and the heat transfer rate can be controlled by the application of a strong magnetic field. They also showed the influence of the body acceleration parameteron the axial velocity, pressure and temperature distributions.

To the best of our knowledge, radiative heat transfers of non-Newtonian blood flow through a

narrow artery in the presence of magnetic field, slip conditions, viscous dissipation have not been investigated before. Also, the study will be beneficial to surgeons and medical practitioners as a whole during surgery and therapeutic situations of some diseases.

2.0 Mathematical Formulation

We considered the blood flow in small artery to be steady, two-dimensional and fully developed, where the flowing blood is assumed to be non-Newtonian portrayed by the Casson fluid model. The flow is subjected to low pressure gradient from the action of the heart, buoyancy force caused by temperature difference between the wall and the inside medium, radiation and slip forces. A magnetic field of strength B_0 is applied normal to the flow. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field is negligible. For the mathematical analysis that follows, the Cartesian coordinates (x, y) is employed, where $x-$ axis is taken be along the horizontal entrance/opening of the artery and $y-$axis is along the transverse direction to x axis (see Figure 1 below).

Consequent on the above assumptions, the equations that govern the flow of blood for the present problem may be written as

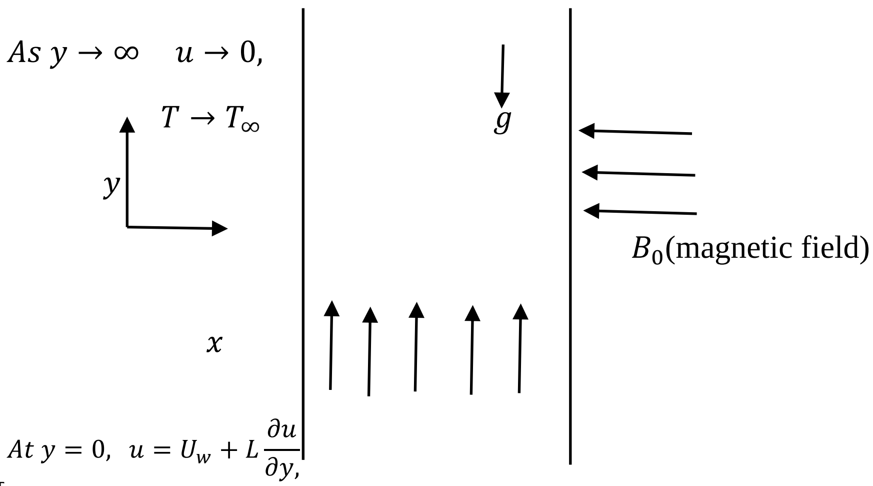

Figure1: Sketch of F

$v = 0, T = T_w + T_s$ (1)

Direction of flow through the artery

$$u\frac{}{\partial x} + v\frac{}{\partial y} = -\frac{}{\rho}\frac{}{dx} + \frac{}{\rho}\left(1 + \frac{}{\beta}\right)\frac{}{\partial y}\left[\mu\frac{}{\partial y}\right] + g\beta_T(T - T_\infty) - \frac{u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_B(T)}{\rho C_p}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{(\rho C_p)_{hnf}}\frac{\partial q_r}{\partial y} \tag{3}$$

Boundary conditions

$$u = u_w + L\frac{\partial u}{\partial y}, v = 0,\ T = T_w + k\frac{\partial T}{\partial y}, \quad at\ y = 0 \tag{4}$$

$$u \to 0\ ,\ T \to T_\infty \qquad as \quad y \to \infty \tag{5}$$

Here, $U_w = ax$ stands for the velocity of the stretching of the blood artery, v denotes the velocity at the artery surface,B_0 represents the applied transverse magnetic field strength, L stands for the proportionality constant of slip velocity, $\beta = \frac{\mu\sqrt{2\pi_c}}{p_y}$ stands for the non-Newtonian material/ Casson parameter, p_y is the fluid yield stress, π_c represents the product of the component of deformation rate for blood, μ is the blood plastic dynamic viscosity, T is the blood temperature, T_w is the arterial wall temperature, T_∞ is the free stream temperature, g is the gravitational acceleration, ρ is the blood density, β_T is the thermal expansion coefficient, k is the blood thermal conductivity, P represents the blood pressure, C_p denotes the specific heat at constant pressure. We assumed that blood is incompressible, suspension of erythrocytes in plasma, has a constant density and a uniform hematocrit (the ratio of red blood cell to the entire blood composition) level. Consequent on these assumptions, we shall follow the ideas of Animasaun (2015) and Ajayi *et al.* (2017) to take the blood variable viscosity to be of the form

$$\mu(T) = \mu^*[1 + b(T_w - T)] \quad (6)$$

Using the Roseland approximation (Ngufor, 2016; Misra and Sinha, 2013) for thermal radiation in optically thick layer, the radiative heat flux for blood taken to be

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \approx \left(-\frac{16\sigma^* T_w^3}{3k^*}\frac{\partial^2 T}{\partial y^2}\right) \quad (7)$$

where σ^* is the Stefan Boltzmann constant, k^* is the mean absorption coefficient.

Introducing the following established variables

$$\eta = y\sqrt{\frac{a}{\vartheta}}, \qquad f(y) = \frac{\psi}{x\sqrt{\vartheta a}} \text{ and } \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (8)$$

And substituting the terms of equation (8) into (1) – (5), continuity equation automatically satisfied the remaining equations reduces to

$$\left(1 + \frac{1}{\beta_c}\right)[1 + (1-\theta)\xi]\frac{d^3 f}{d\eta^3} + G\xi\theta - M\frac{df}{d\eta} - \frac{df}{d\eta}\frac{df}{d\eta} + f(\eta)\frac{d^2 f}{d\eta^2} + \lambda - \xi\left(1 + \frac{1}{\beta_c}\right)\frac{d^2 f}{d\eta^2}\frac{d\theta}{dy} = 0 \quad (9)$$

$$\left(\frac{3 + 4R}{3}\right)\frac{d^2\theta}{d\eta^2} + P_r f(\eta)\frac{d\theta}{d\eta} + \left(1 + \frac{1}{\beta_c}\right)[1 + (1-\theta)\xi]P_r E_c\frac{d^2 f}{d\eta^2}\frac{d^2 f}{d\eta^2} = 0 \quad (10)$$

with boundary conditions

$$\frac{df}{d\eta} = 1 + \delta\frac{d^2 f}{d\eta^2}, \qquad (\theta - 1) = \gamma\frac{d\theta}{d\eta}, \qquad f(\eta) = 0, \qquad \text{for } y = 0$$

$$\frac{df}{d\eta} \to 0, \;\; \theta \to 0 \qquad as \quad y \to \infty \quad (11)$$

Here, pressure gradient parameter $= -\frac{1}{\rho x a^2}\frac{\partial P}{\partial x}$, temperature dependent viscosity parameter $\xi = b(T_w - T_\infty)$, magnetic parameter $M = \frac{\sigma B_0^2}{a\rho}$, buoyancy parameter $G = \frac{g\beta_T}{a^2 x b}$, Prandtl number $P_r = \frac{(\rho C_p)_f \vartheta}{K_f}$, Radiation parameter $R_a = \frac{4\sigma^* T_\infty^3}{k^* k}$, Eckert number $E_c = \frac{a^2 x^2}{C_p}$, velocity slip parameter $\delta = L\frac{U_w^{\frac{1}{2}}}{\vartheta^{\frac{1}{2}}}$, thermal slip parameter $\gamma = k\frac{a^{\frac{1}{2}}}{\vartheta^{\frac{1}{2}}}$.

The physical qualities of engineering interest in this study are the skin friction coefficientc_f and rate of heat transfer or local Nusselt number Nu_x which are defined respectively as

$$C_f = \frac{f''(0)}{\left(1+\frac{1}{\beta}\right)\sqrt{Re_x}} \quad , Nu_x = \frac{-\theta'(0)}{\sqrt{Re_x}} \tag{12}$$

where Reynold number $R_e = \frac{x^2 a}{\vartheta}$

3. Numerical Solution

The set of non-linear coupled differentials equations (9) – (11) are first reduced to a system of first order ordinary differential equations. This is because the Runge Kutta method can only integrate ordinary differential equations of first order. The resulting system of ordinary differential equations are thereafter solved numerically by using the default Matlab bvp5c code that implements an implicit Runge-Kutta formula. It is unpractical to solve the above equations on an infinite interval, thus, we shall impose a finite boundary at a finite point $\eta = 10$. The system then computes the results.

4. Results and Discussion

Numerical study to examine the variation of various physical parameters of major physiological significance has been undertaken and the results are all exhibited through figures and table.

Figure 2 elucidates the influence of radiation parameter over the temperature profile. The figure depicts that thermal radiation absorption causes an enhancement in blood temperature. This is

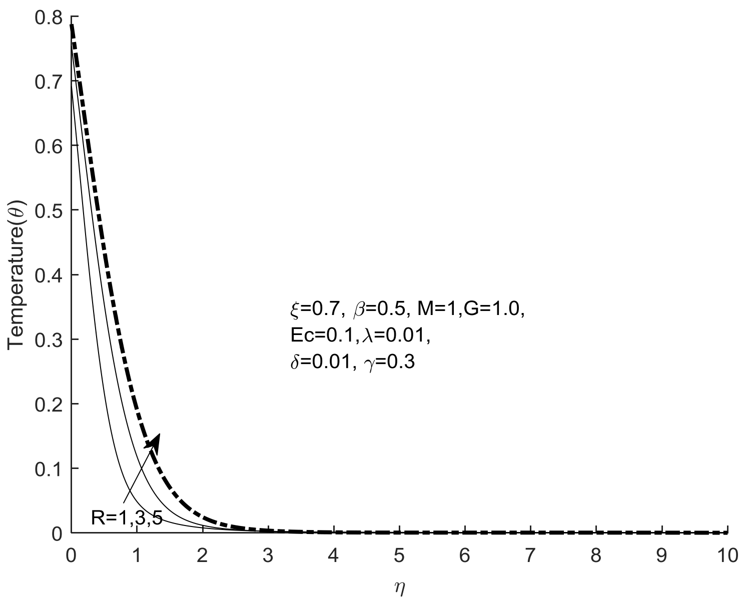

Figure 2: Variation in Radiation parameter(R) with Temperature profile

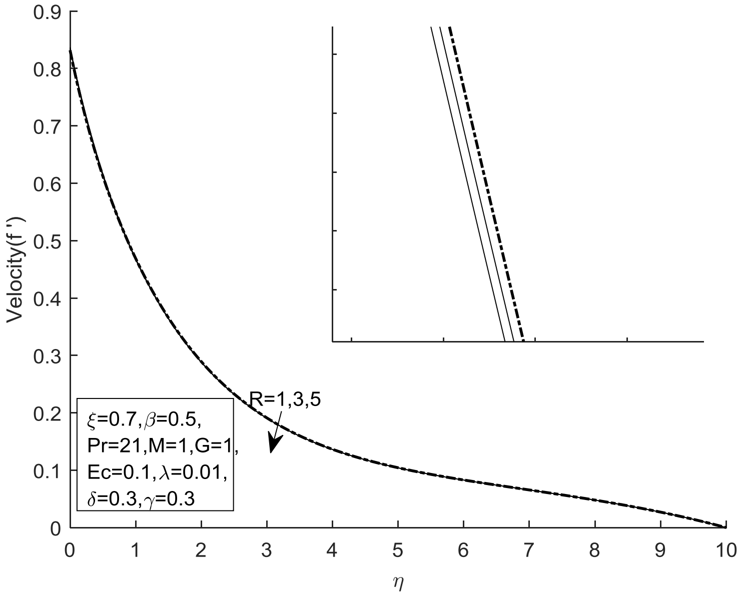

Figure 3: Variation in Radiation parameter (R) with Velocity

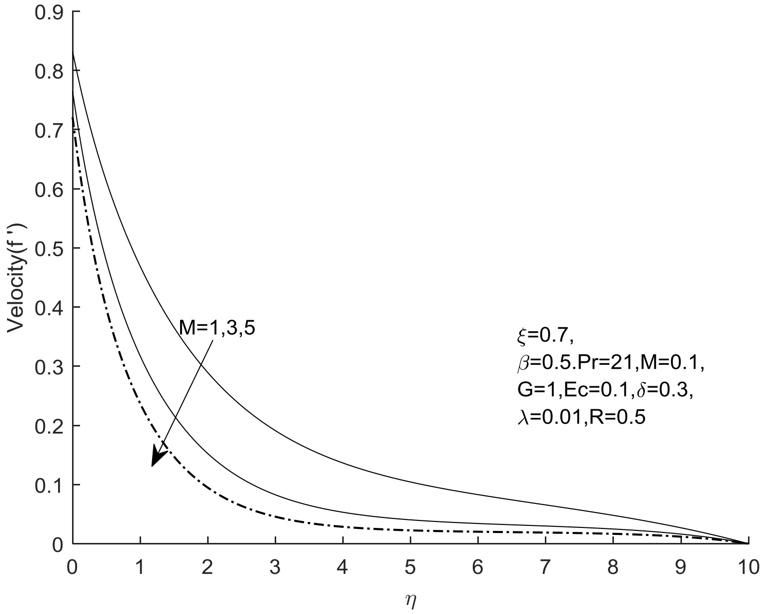

Figure 4: Variation in magnetic parameter (M) with velocity

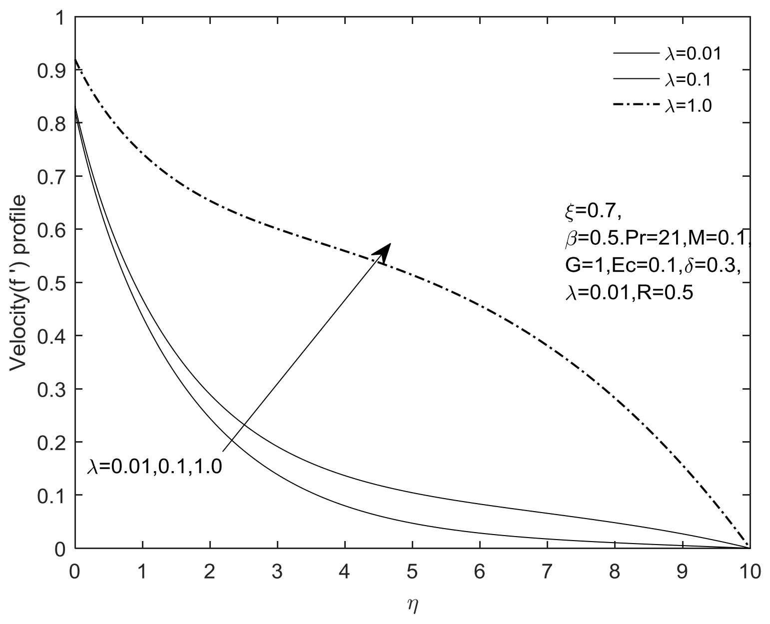

Figure 5: Variation in pressure gradient parameter(λ) with velocity

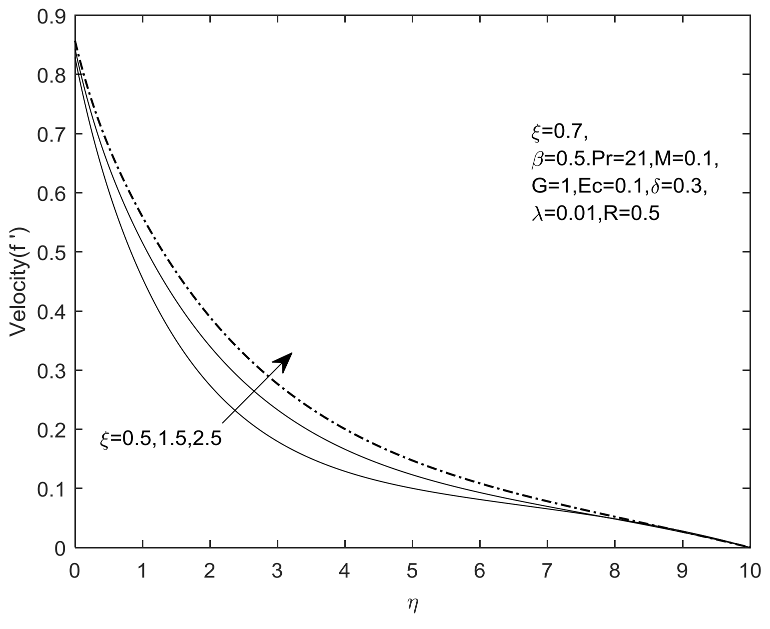

Figure 6: Variation in variable viscosity parameter (ξ) with velocity

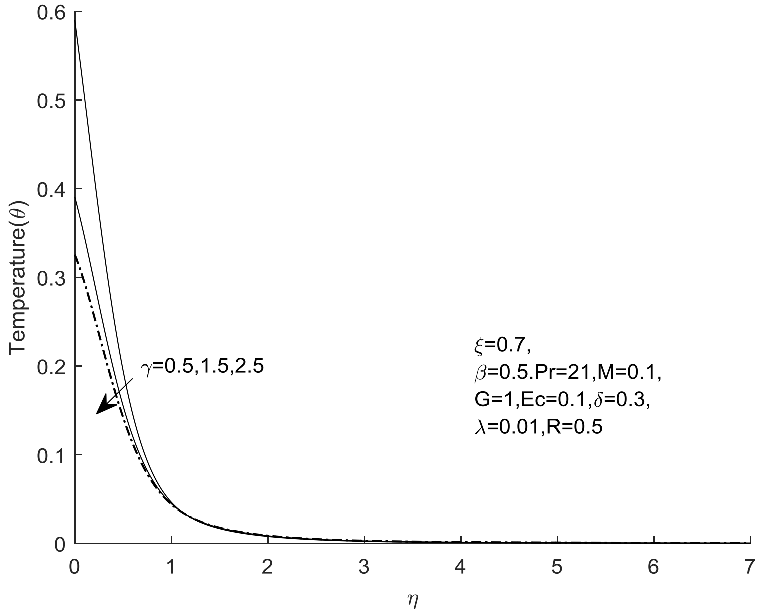

Figure 7: Variation in thermal slip parameter (γ) with Temperature

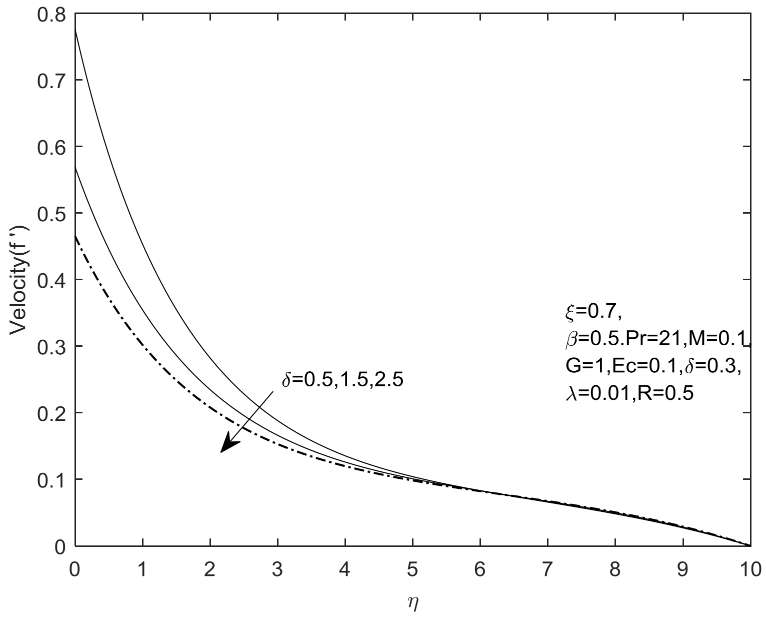

Figure 8: Variation in velocity slip parameter (δ) with velocity

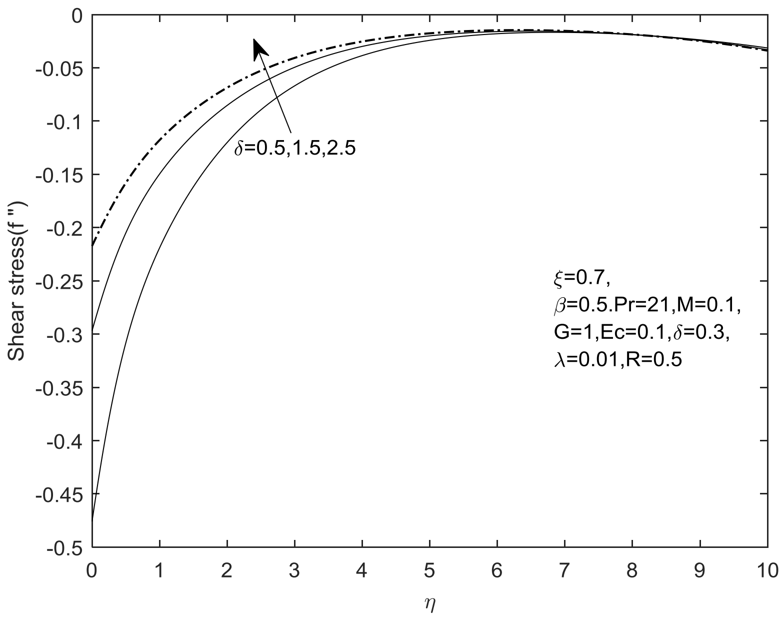

Figure 9: Variation in velocity slip parameter (δ) with Shear stress

Table 3.2: Numerical results from different values of the controlling parameters and corresponding local Skin friction, Nusselt number (when $\lambda = 0.01, \epsilon = 0.7, M = 0.1, R = 0.5$**)**

ξ	γ	δ	G	P_r	β	E_c	$\left(1+\frac{1}{\beta}\right)f''(0)$	$-\theta'(0)$
0.3	0.3	0.3	0.1	21	0.5	0.01	-0.426146924	1.217526191
0.5							-0.416842958	1.214607687
0.7							-0.4083374317	1.211784491
0.7	0.3	0.3	1	21	0.5	0.01	-0.394972415	1.211341763
			2				-0.380416066	1.210948950
			3				-0.366174465	1.210665893
0.7	0.5	0.3	0.1	21	0.5	0.01	-0.401061159	0.968325708
	1.0						-0.392193024	0.663357634
	1.5						-0.388113579	0.519097120
0.7	0.3	0.3	0.1	18	0.5	0.01	-0.424878558	1.214560614
				20			-0.449567747	1.217553022
				22			-0.467253497	1.218956048
0.7	0.3	0.3	0.1	21	0.5	0.1	-0.408374317	1.211784491
						0.5	-0.453705159	0.894998360
						1.0	-0.556519783	0.050000644

0.7	0.3	0.5	0.1	21	0.5	0.01	-0.3632433797	1.198932244
		1.0					-0.287780076	1.170138163
		1.5					-0.2130675338	0.969243508

cancer cells associated with tumor with minimal injury to normal tissues. The influence of radiation on fluid velocity is shown in Figure 3. It depicts that radiation slightly retards the blood velocity. The effect of Lorentz force on velocity profile can be observed in Figure 4.

The figure reveals that velocity gets decelerated as magnetic parameter (M) increases. The Physics behind this, is that the Lorentz force forms an interaction of magnetic and electric fields which triggers the opposite behavior to fluid motion and a slowdown of motion results. This is in good agreement with Figure 21 as reported by Ajayi et al. (2017) and Fig.2 in Misra and Sinha (2013). The influence of pressure gradient parameter on the fluid velocity is depicted in Figure 5. It is revealed that pressure gradient inflates blood velocity. Figure 6 shows the graphical representation of the velocity profile for different values of variable viscosity parameter (ξ). The figure showed that viscosity parameter increases the rate of blood flow. The reason for this behavior is that an increase in viscosity parameter leads to a reduction in fluid viscosity which corresponds to a corresponding rise in fluid flow. In figure 7, thermal slip parameter deflates fluid temperature. This is in harmony with Figure 8 as reported by Misra and Sinha (2013). The variation in blood velocity for increase in velocity slip is clucidated in Figure 8. The figure reveals that the velocity slip parameter causes a retardation in blood velocity. This is because, as velocity slip parameter increases, the fluid experiences less drag. This supports figure 1 reported by Singh and Makinde (2013). However, velocity slip parameter enhances the shear stress profile as depicted in Figure 9.

In Table 1, variable viscosity increases the skin friction coefficient but decreases the Nusselt number. Buoyancy parameter also increases the skin friction but reduces the heat transfer coefficient. Thermal slip parameter also increases the skin friction coefficient and decreases the Nusselt number. Velocity slip parameter and Prandtl number inflates both the shear stress and Nusselt number. The increase in Nusselt number for increase in Prandtl number is in good agreement with figure 10 of Misra and Sinha(2013). This hangs on the fact thatas blood Prandtl number increases, the thermal conductivity reduces, thus the heat conduction capacity reduces. As a result, the thermal boundary layer thickness gets reduced and the heat transfer rate at the vessel wall get enhanced. As such, blood temperature decreases while Nusselt number increases. While Eckert number deflates both skin friction coefficient and Nusselt number.

5. Conclusion

In the present investigation of radiative heat transfer of non-Newtonian blood flow through a

narrow artery in the presence of magnetic field, analytical expressions of key variables are obtained and shown graphically. This study can be of help in pathological situations. Radiation increase temperature and so can help to increase in blood temperature which can be used to kill cancer cells. Pressure gradient inflates blood velocity. Thermal slip parameter transfer rate.

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