International Conference of Sciences, Engineering & Environmental Technology (ICONSEET), 6(29): 223 – 235, 2021
ISSN 0794-9650
www.repcomseet.com

STOCHASTIC ANALYSIS OF MULTIPATH FADING WITH PHASE ERROR OVER WEIBULL FADING CHANNEL IN EQUAL GAIN DIVERSITY

[1]Agbolade, J. O., [2]Ojedokun, I. A., [1]Adebayo, A. K. [1]Afolalu, O. F., and [1]Bamikefa, I. A.
[1]Department of Electrical and Electronic Engineering, School of Engineering, Federal Polytechnic, Ede, Nigeria
[2]Department of Electrical and Electronic Engineering, Faculty of Engineering, Federal University Otuoke, Nigeria
yemilade2013@gmail.com

Abstract: In any realistic wireless communication scenario, multipath fading is a predominant effect which could be worsened by Phase Error (PE). The combine effect of multipath fading and PE encountered in this medium degrades the signal. Weibull distribution (fading) channel has been found to be very useful and appropriate to model wireless communication channel. In this paper we present the analysis of the Wireless channel corrupted by the PE at the receiver using Weibull and Tichonov distributions. The closed-form expressions were obtained using the Probability Density Function (PDF) method over Equal Gain Combining (EGC) for Outage Probability (P_o) and Bit Error Rate (BER). The results obtained show that the higher the PE the higher the outage probability and the higher the BER of the received signal. It is therefore necessary for effective signal reception the PE must be kept as low as possible within the allowable value. Both simulation and theoretical results were in agreement.

Key Words: Probability Density Function, Equal Gain Combining, Outage Probability, Bit Error Rate, Phase Error, Weibull fading.

A. Introduction

Wireless Communication (WC) has become one of the heartbeats of the world economy. However, it suffers from multipath fading and phase error. These are due to the nature of channel of propagation and the phase lock loop (PLL) which is used to recover the carrier phase. These lead to imperfect recovery of the transmitted signal at the receiver leading to noise and interference. WC is modeled by different distributions that describe the statistical behavior of the channel depending on the nature of the wireless environment. Such distribution includes but not limited to Rayleigh, Rician, Nakagami and Weibull.

The Weibull distribution has been proposed as an appropriate statistical fading model to describe fading channels. Much emphasis has been placed on the three most common fading models (Rayleigh, Rician and Nakagami) to characterize multipath propagation medium. The versatility of Weibull distribution has been demonstrated in several scientific fields and much recently in wireless communications. [13] established that Weibull model exhibits a perfect fit to experimental fading channel measurement for both indoor and outdoor environment.

A lot of work has considered Weibull fading channel analysis using performance metrics in Physics. This fading channel was recommended for theoretical studies by the IEEE Vehicular Technology Society Committee on Radio Propagation [1]. In [6], Weibull distribution provides optimum for Digital Enhanced Cordless Telecomm (DECT) systems operating at 1.89GHz. This was followed by the work in [26], where several experiments for Global System for Mobile Communication (GSM) network, operating at 900MHz and confirmed its usefulness as a fading model for outdoor systems. In [21], channel capacity and second-order statistics are analyzed over Weibull fading channel. In [14], development of a novel channel model was carried out, the model is a synthesizer for generating rain attenuation time series for satellite links operating at 10GHz and above. The channel model is a modification of Masenge-Bakken (M-B) model because it generates rain attenuation time series that follows the Weibull distribution.

In [24], development of a new approximate closed form expression (distribution) and performance analysis of composite Weibull/Log-Normal fading channel was done. In [10], a comparative simulation analysis on the performance of Low-Density Parity Check (LDPC) coded communication systems over Weibull fading channels was attempted. In [7], there was a development of a method for approximating the probability distribution of a sum of independent and identical Weibull RV which was adopted to analyze the performance of Equal Gain Combiner (EGC) receiver over non-identical Weibull fading channel.

Among other numerous wireless communication applications of this fading channel that has been attempted includes the work of [8], [15] and [11]. It is imperative however, to observe that no significant study has been carried out on the performance of WC channel over Weibull fading channel that has been corrupted by both multipath fading and phase error. Therefore, our proposed models aim to fill this gap in knowledge. The main objective is to developed statistical model to obtain P_o and BER for EGC over multipath fading which has been model as Weibull fading distribution and Phase Error which is Tichonov distributed.

B. Channel Modeling

In considering a Weibull fading channel, Cvetkovic et al (2011) define the received signal y(t) as

$$y(t) = s(t)\exp\left[j\left(2\pi f_c t + \varphi(t) + \gamma(t)\right)\right] + n(t) \qquad (1)$$

where s(t) is the Weibull distributed random variable with its own pdf, n(t) is the AWGN noise which is a random variable (RV) with mean zero and variance $N_0/2$, $\varphi(t)$ is the information bearing phase, f_c is the carrier frequency and $\gamma(t)$ is the random phase uniformly distributed in [0, 2π].

C. Weibull fading distribution

Weibull distribution is used to model multipath wave propagation in a non- homogeneous environment. Weibull distribution is suitable to characterize the mobile radio systems operating in the frequency range of 800 to 900 MHZ for both indoor and outdoor environments [22]. The PDF of Weibull distribution, ($P_{\alpha W}(\alpha)$) is given by [22].

$$P_{\alpha W}(\alpha) = \beta\left(\frac{\Gamma\left(1+\frac{2}{\beta}\right)}{\Omega}\right)^{\frac{\beta}{2}} \alpha^{\beta-1} exp\left[-\left(\frac{\alpha^2}{\Omega}\Gamma\left(1+\frac{2}{\beta}\right)\right)^{\frac{\beta}{2}}\right] \qquad (2)$$

where

β ϵ (0, ∞) is the Weibull fading parameter and $\Omega = \mathrm{E}[\alpha^2]$.

Weibull PDF simplifies to Rayleigh PDF when β = 2 [22].

D. Phase Error

In a multipath fading channel, signal transmitted undergoes random phase shift, in revolving the multiple signals and combining them effectively, the receiver requires the knowledge of the phase and amplitude changes introduced by each path. Therefore, there is a need to know an allowable phase error that would not impact the receiver negatively. An effective combining technique should be able to undo the phase shift of the received signal. Hence, estimation of the phase of the received signal is required. The first order Phase Lock Loop (PLL) is a circuit that estimate the carrier phase but due to Doppler spread, thermal noise and other unwanted signals presence in this circuit the phase is incorrectly estimated in the Carrier Recovery loop (CRL) segment of the circuit, this incorrect carrier recovery of the phase is called the phase error. This inadvertently causes poor performance of the diversity combining receivers [16], [18], [20] and [11]. When the PLL is used for carrier recovery, the random phase error, (PE) follows the Tichonov distribution, ($F_\theta(\theta)$) given by [16] as

$$F_\theta(\theta) = \frac{exp(\rho\cos\theta)}{2\pi I_0(\rho)} \qquad (3)$$

where, ρ is the Carrier Recovery Loop (CRL) SNR

I_0 is the zeroth order modified Bessel function of the first kind

θ is the phase angle

The root mean square (RMS) phase error, σ, is related to CRL, ρ, as $\sigma = 1/\sqrt{\rho}$

In order to guarantee a reasonably coherent reception, the RMS, σ, tracking the signal phase should lie within 2^o – 12^o, [16]. The instantaneous output SNR with phase error in L-branch EGC, B_{egc}, was given by [3] and [4] as

$$B_{egc} = \left(\sum_{l=1}^{L} \sqrt{B}\ \cos\theta_l\right)^2 \quad (4)$$

where B is the instantaneous SNR of the fading channel considered and θ_l is the phase angle of l^{th} branch EGC. Consequently, from this equation, using the PDF based approach to find the PDF of B_{egc}, required the derivation of the PDF of $\sqrt{B}$, θ_l, and $\cos\theta_l$. The PDF of θ_l, which is Tikhonov distributed, $f_{\theta_l}(\theta_l)$ is given in [Najib and Prabhu, 2000] and simplified in [5] as

$$f_{\theta_l}(\theta_l) = \frac{exp-\left(\frac{\rho_l \theta_l{}^2}{2}\right)}{\sqrt{\frac{2\pi}{\rho_l}}} \quad (5)$$

Since the value of PE must be very small for a meaningful signal reception at the receiver, and that $\sin\theta_l \cong \theta_l$ in radian when $1^0 \leq \theta \leq 10^0$ and therefore, [2] showed that

$$f_{\cos\theta_l}(\cos\theta_l) \cong f_{\theta_l}(\theta_l) = \sqrt{\frac{2\rho_l}{\pi}} \frac{exp-\left(\frac{\rho_l \theta_l{}^2}{2}\right)}{\theta_l} \quad (6)$$

Where, ρ_l is the SNR of the Carrier Recovery Loop (CRL) and is related to σ_l, the Phase Error (PE) given by [Lindsey and Chie, (1986)] as.

$$\rho_l = \frac{1}{(\sigma_l)^2} \quad \Rightarrow \quad \sigma_l = \sqrt{\frac{1}{\rho_l}} \quad (7)$$

This implies that the higher the CRL, the lower the value of PE

E. Probability Density Function of Weibull Fading Channel with Phase Error

The Probability Density Function (PDF) of Weibull fading channel is given in [22] as

$$P_\alpha(\alpha) = \beta(\delta)^{\frac{\beta}{2}} \alpha^{\beta-1} exp\left[-(\alpha^2 \delta)^{\frac{\beta}{2}}\right] \quad (8)$$

Where, $\delta = \frac{\Gamma\left(1+\frac{2}{\beta}\right)}{\Omega}$ then

Using Stark and Wood, (2002) the power of the PDF, $f_{\sqrt{\alpha}}(\sqrt{\alpha})$, for Weibull fading distribution is derived as

$$f_{\sqrt{\alpha}}(\sqrt{\alpha}) = 2\alpha^{\beta-\frac{1}{2}} \beta(\mathrm{h})^{\frac{\beta}{2}} exp-(\mathrm{h}\alpha^2)^{\frac{\beta}{2}} \quad (9)$$

The PDF of the product $C = \theta_l \times \sqrt{\alpha}$ is derived using equation (6) and (9), random variable transformation in [25], identities in [12] and [19] and substituting for $\theta_l = \frac{C}{\sqrt{\alpha}}$, the combined PDF of Weibull fading channel with PE, after some mathematical simplifications is

$$f_C(C) = \frac{4\beta\delta^{\frac{\beta}{2}}\sqrt{2\rho_l}}{C\sqrt{\pi}}\int_0^\infty \alpha^{\beta-\frac{1}{2}} \times exp\left(-\delta^{\frac{\beta}{2}}\left(\sqrt{\alpha}\right)^{2\beta} - \frac{\rho_l C^2}{2}\left(\sqrt{\alpha}\right)^{-2}\right) d\sqrt{\alpha} \qquad (10)$$

Solving equation (10) is highly demanding, but not insurmountable, the closet identity from the table of integrals by [12] requires that β, the Weibull parameter, to be one. Hence, for $\beta = 1$, therefore, $\delta = \frac{\Gamma\left(1+\frac{2}{\beta}\right)}{\Omega} = \frac{2}{\Omega}$ in equation (10) after substituting for δ and β is

$$f_C(C) = \frac{4\left(\frac{2}{\Omega}\right)^{\frac{1}{2}}\sqrt{2\rho_l}}{C\sqrt{\pi}}\int_0^\infty \alpha^{\frac{1}{2}} \times exp\left(-\left(\frac{2}{\Omega}\right)^{\frac{1}{2}}\left(\sqrt{\alpha}\right)^2 - \frac{\rho_l C^2}{2}\left(\sqrt{\alpha}\right)^{-2}\right) d\sqrt{\alpha} \qquad (11)$$

Solving this equation, we make use of the identity in [12], the solution of the integral in equation (11), therefore, is

$$f_C(C) = \frac{8}{C}\sqrt{\frac{\rho_l}{\pi\Omega}}\left(\frac{\rho_l C^2}{2}\sqrt{\frac{\Omega}{2}}\right)^{\frac{1}{2}} K_{\frac{2}{2}}\left(2\sqrt{\frac{\rho_l C^2}{2}\sqrt{\frac{2}{\Omega}}}\right) \qquad (12)$$

where $K_V(z)$ is given by [19] as $K_V(z) = \sqrt{\frac{\pi}{2z}}e^{-z}$ therefore, equation (12) after substituting $K_V(z)$ and simplifying becomes,

$$f_C(C) = \frac{2^{\frac{9}{4}}(\rho_l)^{\frac{3}{4}}}{C^{\frac{1}{2}}\Omega^{\frac{3}{8}}} e^{-2^{\frac{3}{4}}C(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}} \qquad (13)$$

It is required that the PDF of the sum of path $C = C_1 + C_2 + \cdots \ldots \ldots, C_L$, where $l = 1, 2, \ldots \ldots \ldots L$ is found by using the Fourier Transform (FT) approach which is otherwise known as the characteristics function approach given in [25] and [12] as

$$\emptyset_{C_l}(\omega) = \int_0^\infty f_{C_l}(C_l)\, e^{j\omega\, C_l} dC_l \qquad (14)$$

$$f_C(C) = N\int_0^\infty C_l^{-\frac{1}{2}} e^{-\left(2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}} - j\omega\right)C_l} dC_l \qquad (15)$$

Where, $N = \frac{2^{\frac{9}{4}}(\rho_l)^{\frac{3}{4}}}{\Omega^{\frac{3}{8}}}$

In [12], the possible solution to the definite integral of the form in equation (13) in one of its identities was analysed. Therefore, by substitution, comparison and equating indices

$$\emptyset_{C_l}(\omega) = N\left(2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}} - j\omega\right)^{-\left(\frac{1}{2}\right)}\Gamma\left(\frac{1}{2}\right) \qquad (16)$$

By using the CHF in [Stark and Wood, 2002; Gradshteyn and Ryzhik, 2007], the inverse of FT and substituting for $C = \sqrt{B_e}$ with some mathematical operations

$$f_C(C) = \frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L}{2\pi}\int_0^{\infty}\left(2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}} - j\omega\right)^{-\frac{1}{2}L} e^{-j\omega C}\, d\omega \tag{17}$$

Similarly using the table of integrals by [12], the possible solution to the definite integral of the form in equation (17) is

$$f_C(C) = \frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L C^{\frac{1}{2}L-1} e^{-2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}} C}}{\Gamma\left(\frac{1}{2}L\right)} \tag{18}$$

$$f_{B_e}(B_e) = \frac{1}{\Gamma\left(\frac{1}{2}L\right)}\left(\frac{2^{\frac{9}{4}}(\rho_l)^{\frac{3}{4}}}{\Omega^{\frac{3}{8}}}\Gamma\left(\frac{1}{2}\right)\right)^L \left(\sqrt{B_e}\right)^{\frac{1}{2}L-1} exp\left(-2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}\sqrt{B_e}\right) \tag{19}$$

Equation (19) is the PDF of the output of EGC with PE over Weibull fading channel with Weibull parameter, $\beta = 1$.

F. Outage Probability of EGC with PE over Weibull Fading Channel

Simon and Alouini (2005) define to outage probability, (P_o), as given below

$$P_O = \int_0^{B_{th}} f_{B_e}(B_e)\, dB_e$$

Therefore, using equation (19) the outage probability is given by

$$= \frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L}{\Gamma\left(\frac{1}{2}L\right)}\int_0^{B_{th}}\left(\sqrt{B_e}\right)^{\frac{1}{2}L-1} exp\left(-2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}\sqrt{B_e}\right) dB_e \tag{20}$$

for simplification let $Y = \frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L}{\Gamma\left(\frac{1}{2}L\right)}$, and $n = 2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}$, therefore, equation (20) simplifies to

$$P_O = Y\int_0^{B_{th}}\left(\sqrt{B_e}\right)^{\frac{1}{2}L-1} exp\left(-n\sqrt{B_e}\right) dB_e \tag{21}$$

by variable transformation, $\sqrt{B_{egc}} = x, \Rightarrow dB_{egc} = 2x dx$, equation (21) is

$$P_O = 2Y\int_0^{B_{th}} (x)^{\frac{1}{2}L} e^{-nx}\, dx \tag{22}$$

The solution of this type of integral is given by [12], therefore, comparing and equating indices, the solution of equation (22) is

$$P_o = 2Yn^{-\left(\frac{1}{2}L+1\right)}\gamma\left(\left(\frac{1}{2}L+1\right), nB_{th}\right) \quad (23)$$

The incomplete gamma function $\gamma(\,.\,,\,.\,)$ is expressed in series form by [19] and given as $\gamma(1+a,x) = a!\left[1 - e^{-x}\left(\sum_{b=0}^{a}\frac{x^b}{b!}\right)\right]$, replacing this in equation (23)

$$P_o = 2Yn^{-\left(\frac{1}{2}L+1\right)}\left(\frac{1}{2}L\right)!\left[1 - e^{-(nB_{th})}\left(\sum_{b=0}^{\frac{1}{2}L}\frac{(nB_{th})^b}{b!}\right)\right] \quad (24)$$

Substituting for Y and n

$$P_o = 2\frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L}{\Gamma\left(\frac{1}{2}L\right)}\left(2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}\right)^{-\left(\frac{1}{2}L+1\right)}\left(\frac{1}{2}L\right)!$$

$$\times\left[1 - exp\left(-\left(2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}\right)B_{th}\right)\left(\sum_{b=0}^{\frac{1}{2}L}\frac{\left(\left(2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}\right)B_{th}\right)^b}{b!}\right)\right]$$

(25) [22] gives an expression for the ratio of instantaneous SNR and average SNR as $\frac{B}{\bar{B}} = \frac{a^2}{\Omega}$ therefore, $\bar{B} \cong \Omega$, substituting N, $\bar{B}$, and equation (7) in equation (25) we have

$$P_o = 2\frac{\left(4.76\times\Gamma\left(\frac{1}{2}\right)\right)^L\left(\frac{1}{2}L\right)!}{\Gamma\left(\frac{1}{2}L\right)(\bar{B})^{\frac{1}{4}(L-1)}(\sigma_l)^L}\left[1 - exp\left(-\left(\frac{1.68\times B_{th}}{(\bar{B})^{\frac{1}{4}}\sigma_l}\right)\right)\left(\sum_{b=0}^{\frac{1}{2}L}\frac{1}{b!}\left(\frac{1.68\times B_{th}}{(\bar{B})^{\frac{1}{4}}\sigma_l}\right)^b\right)\right] \quad (26)$$

Equation (26) is the outage probability at the output of the EGC combiner in the presence of PE over Weibull fading channel.

G. Bit Error Rate of EGC with Phase Error over Weibull Fading Channel

The BER is given by [22], as

$$P_E = \int_0^\infty Q\left(a\sqrt{2gB_e}\right)f_{B_e}(B_e)\,dB_e \quad (27)$$

Where, $Q(.)$ is the Marcum Q function given in terms of complimentary error function in Olver, *et al.* (2010), but the complimentary error function was express in series form by [12], combining the two, $Q(.)$ after some mathematical operation is

$$Q\left(a\sqrt{2gB_e}\right) = \frac{1}{2} - \frac{2}{\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{(-1)^m(a\sqrt{g})^{2m+1}}{m!(2m+1)}\left(\sqrt{B_e}\right)^{2m+1} \quad (28)$$

BER equation is obtained from equation (27) and (28), where for BPSK $a = 2,\ \ g = \sin^2\frac{\pi}{2}$, substituting equations (19) and (28) in equation (27), the BER over the Weibull fading channel is

$$P_E = \int_0^\infty\left[\frac{1}{2} - \frac{2}{\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{(-1)^m(a\sqrt{g})^{2m+1}}{m!(2m+1)}\left(\sqrt{B_{egc}}\right)^{2m+1}\right]$$

$$\times\frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L}{\Gamma\left(\frac{1}{2}L\right)}\left(\sqrt{B_e}\right)^{\frac{1}{2}L-1}exp\left(-2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}}\sqrt{B_e}\right)dB_e \quad (29)$$

let $X = \frac{2}{\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{(-1)^m(a\sqrt{g})^{2m+1}}{m!(2m+1)}$, $Y = \frac{N^L\left(\Gamma\left(\frac{1}{2}\right)\right)^L}{\Gamma\left(\frac{1}{2}L\right)}$, and $n = 2^{\frac{3}{4}}(\rho_l)^{\frac{1}{2}}(\Omega)^{\frac{-1}{4}} = \frac{1.68}{(\bar{B})^{\frac{1}{4}}\sigma_l}$

Therefore, equation (29) in a simplified form is

$$P_E = \frac{1}{2}Y\int_0^{\infty}\left(\sqrt{B_e}\right)^{\frac{1}{2}L-1}e^{-n\sqrt{B_e}}dB_e - XY\int_0^{\infty}\left(\sqrt{B_e}\right)^{2m+\frac{1}{2}L}e^{-n\sqrt{B_e}}dB_e \qquad (30)$$

By variable transformation and some mathematical operations with the aid of table of integrals by [12], the integrals in equation (30) have a solution of the form

$$P_E = Y(n)^{-\left(\frac{1}{2}L+1\right)}\Gamma\left(\frac{1}{2}L+1\right) - 2XY(n)^{-\left(2m+\frac{1}{2}L+2\right)}\Gamma\left(2m+\frac{1}{2}L+2\right) \qquad (31)$$

$$P_E = \frac{1}{\Gamma\left(\frac{1}{2}L\right)}\left(\frac{4.76\Gamma\left(\frac{1}{2}\right)}{(\sigma_l)^{\frac{3}{2}}(\bar{B})^{\frac{3}{8}}}\right)^L\left(\frac{1.68}{(\bar{B})^{\frac{1}{4}}\sigma_l}\right)^{-\left(\frac{1}{2}L+1\right)}\Gamma\left(\frac{1}{2}L+1\right)$$

$$-\frac{4}{\Gamma\left(\frac{1}{2}L\right)\sqrt{\pi}}\left(\frac{4.76\Gamma\left(\frac{1}{2}\right)}{(\sigma_l)^{\frac{3}{2}}(\bar{B})^{\frac{3}{8}}}\right)^L\sum_{m=0}^{\infty}\frac{(-1)^m(a\sqrt{g})^{2m+1}}{m!(2m+1)}\left(\frac{1.68}{(\bar{B})^{\frac{1}{4}}\sigma_l}\right)^{-\left(2m+\frac{1}{2}L+2\right)}\Gamma\left(2m+\frac{1}{2}L+2\right) \qquad (32)$$

H. RESULTS AND DISCUSSION

Figures 1 and 2 show the BER results in Weibull fading channel. The graph show that the Weibull model is not effective in predicting the complete behaviour of signal transmitted in this channel as it gives a total attenuation of the signal with the high BER observed with Weibull shape parameter equals 1. The model has failed with the high value of BER which is not practicable. Therefore, in Figure 1, the BER results at PE (pha) of 3^o and 12^o for L of 2 and SNR of 4 dB were 6.77×10^4 and 4.33×10^6, respectively, while at L of 8, the BER results were 3.95×10^4 and 2.53×10^6, respectively. At SNR of 18 dB the BER results were 2.19×10^5 and 1.4×10^7, respectively, at L of 2, while at L of 8, 1.22×10^5 and 7.82×10^7 were recorded as BER results.

Figure 2 shows the graph of BER versus PE, at a constant SNR of 2 dB, the Weibull fading channel shows that at PE (pha) of 4^o and L of 2 and 8, the BER results were 3.37×10^5 and 5.57×10^4, respectively, while at PE (pha) of 18^o, the P_E results were 3.07×10^7 and 5.07×10^6, respectively. These results over Weibull fading channel show the ineffectiveness of the model but actually describe what should be expected. The results show that without diversity the received signal strength will be very low and that there could be a complete loss of signal. The research also shows that as the PE increases, the BER results also increase indicating a poor signal reception. In Figure 3 the P_o result at PE of 2^o were 2.24×10^{-4} and 2.49×10^{-13} for L of 2 and 8, respectively. While at 18^o, the results of P_o were 2.71×10^{-2} and 4.36×10^{-6}, respectively. In Figure 4, at L of 2 with SNR of 2 dB and 18 dB, 4.1×10^{-1} and 8.5×10^{-2} were obtained as P_o, respectively, at L of 4, the P_o values were 4.1×10^{-3} and 8.9×10^{-4}, respectively, while at L of 8, the results were 4.1×10^{-7} and 1.62×10^{-8} respectively. These values show that diversity combining leads to an improved received signal strength. The models obtained have shown that as SNR increases, the probability that the signal will be attenuated is very low. Figure 5 shows that in Weibull fading channel at 4 dB SNR, L of 2 and PE of 3^o and 12^o, the P_o results were 1.41×10^{-7} and 2.51×10^{-2}, respectively, but at L of 8, the P_o results obtained were 2.36×10^{-14} and 1.1×10^{-4} for PE (pha) of 3^o and 12^o, respectively. At SNR of 18 dB with L of 2, 1.62×10^{-9} and 8.5×10^{-3} were obtained, respectively as P_o, while at L of 8, the P_o, results were 2.62×10^{-17} and 3.6×10^{-4} for PE (pha) of 3^o and 12^o, respectively. In Figure 6, the P_o were 5.39×10^{-17} and 4.06×10^{-4} respectively, for L of 2. While at L of 8 the P_o results were 1.11×10^{-20} and 3.01×10^{-6} for PE of 13^o and 25^o PE, SNR of 10 dB. Figure 7 indicate the theoretical and simulation results of outage probability versus SNR in Weibull fading channel. These graphs have shown that both theoretical and simulation results were in perfect agreement. Therefore, the theoretical model can be used to predict practical situations where signal communication link is to be planned.

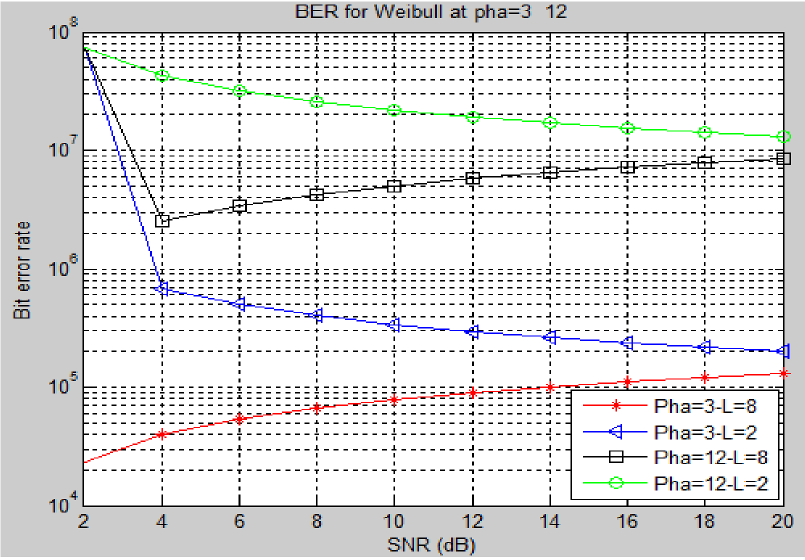

Figure 1: Theoretical Bit Error Rate of BPSK modulated signal versus SNR at PE of 3° and 12° over Weibull fading channel at L of 2 and 8

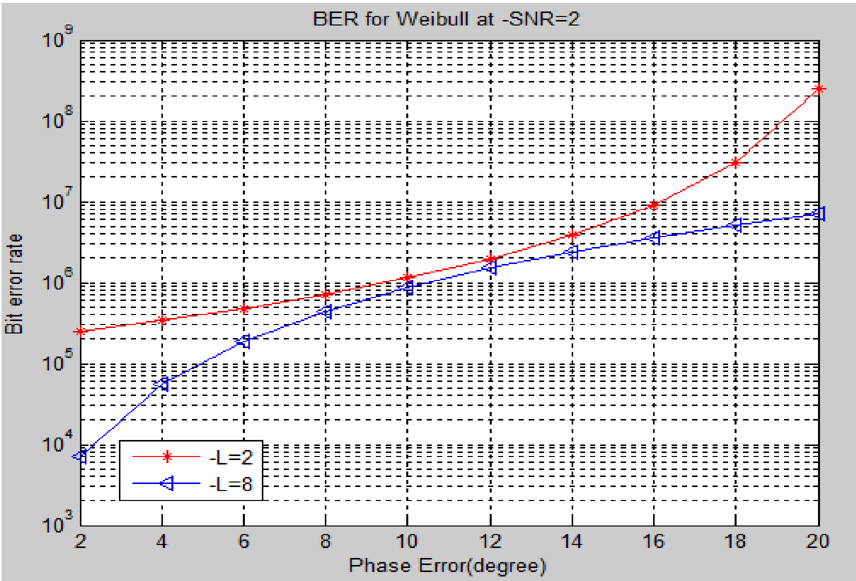

Figure 2: Theoretical Bit Error Rate of BPSK modulated signal versus Phase Error at SNR of 2 dB over Weibull fading channel at L of 2 and 8

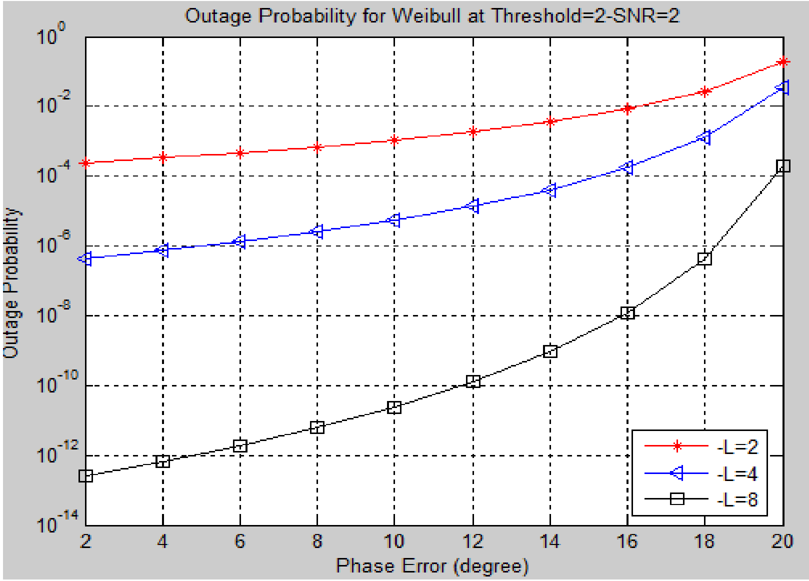

Figure 3: Theoretical Outage Probability versus Phase Error at SNR of 2 dB and a threshold value of 2 dB over Weibull fading channel at L of 2, 4 and 8

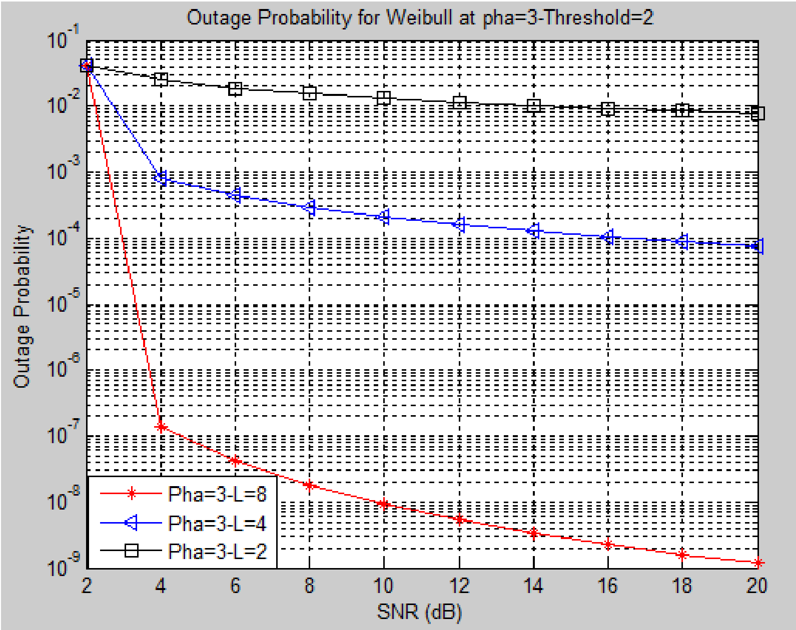

Figure 4: Theoretical Outage Probability versus SNR at a PE (pha) of 3° and a threshold value of 2 dB over Weibull fading channel at L of 2, 4 and 8

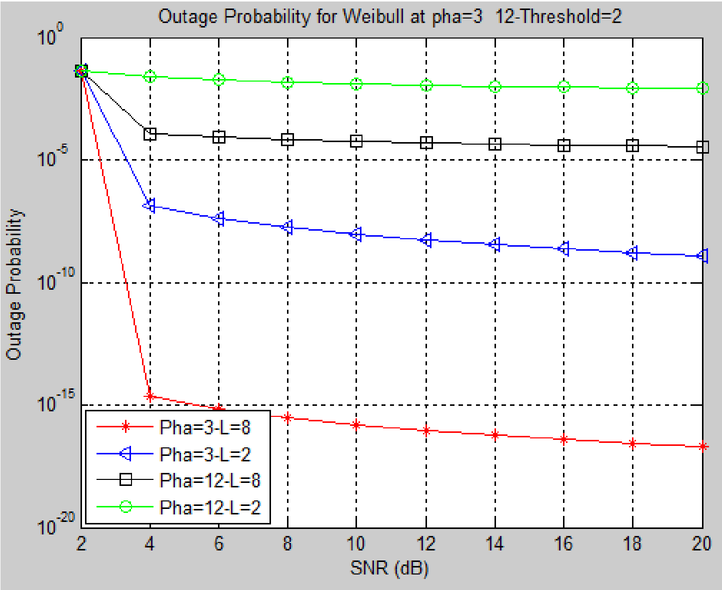

Figure 5: Theoretical Outage Probability versus SNR at a PE (pha)of 3^o and 12^o with threshold value of 2 dB over Weibull fading channel at L of 2 and 8

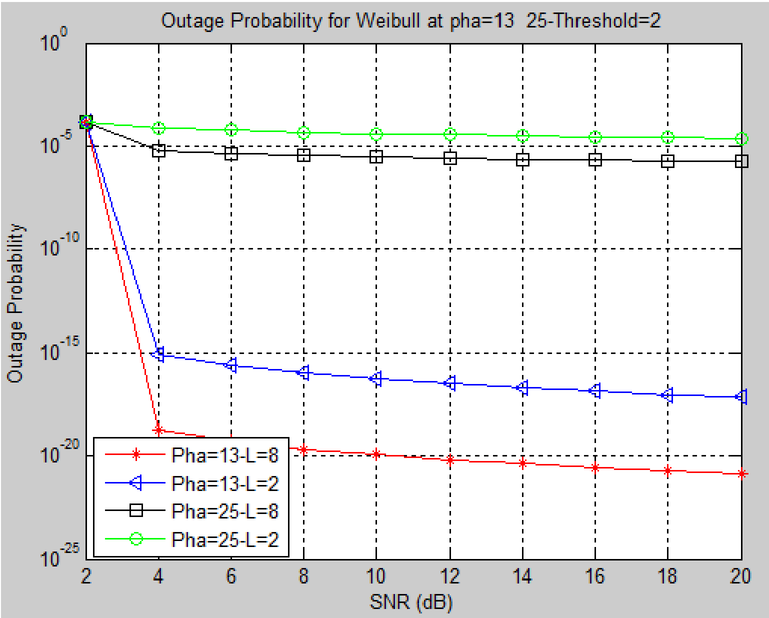

Figure 6: Theoretical Outage Probability versus SNR at a PE (pha) of 13^o and 25^o with threshold value of 2 dB over Weibull fading channel at L of 2 and 8

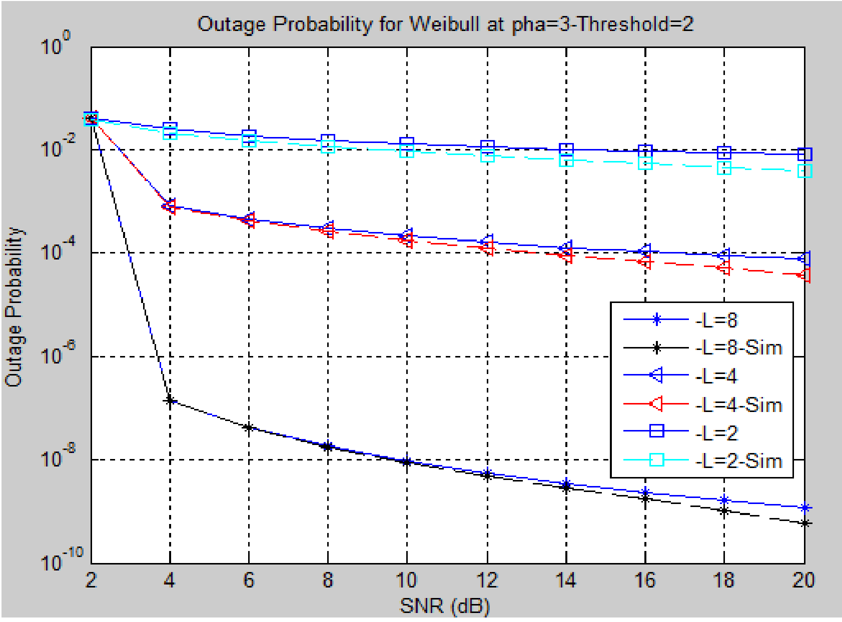

Figure 7: Simulation and Theoretical Outage Probability versus SNR at a PE (pha) of 3^o and a threshold value of 2 dB over Weibull fading channel at L of 2, 4 and 8

I. CONCLUSION

The multipath fading channel was modeled as Weibull distribution fading channel while the phase error was modeled as Tichonov distribution. Statistical analysis was carried out using the properties of the PDF to obtain the PDF of the received signal at the output of EGC. Outage Probability and BER were used to determine the effectiveness of the derived closed-form expression. Computer simulation was carried out and found to be in agreement with the theoretical results. In practical the results have shown that the use of BER as a performance metric is not advisable in this case for Weibull fading channel, it has demonstrated expectations in practical scenario because it shows a complete attenuation of the signal. However the model is suitable for Outage Probability. The results have also shown that the allowable range of PE must not be exceeded to achieve good signal reception.

REFERENCES

[1]. Adawi, N. S., Bertoni, H. L., Child, J. R., Daniel, W. A., Deltra, J. E., Eckert, R. P., and Forest, R. T. (1998). Coverage prediction for mobile radio systems operating in the 800/900MHz frequency range. *IEEE Transactions on Vehicular Technology,* 37(1), 3.

[2]. Aruna, G. and Sahu, P. R. (2012). ABER of Equal Gain Combiner over Correlated Hoyt Fading Channels with Phase Error and Co-Channel Interference, *Annual IEEE India Conference, Indicon, 2012*, India (3):936-940

[3]. Aruna, G. and Sahu, P. R. (2013). ABEP of EGC Receiver over Composite and Non Homogenous Fading Channels with Phase Error and Co-channel, *Interference IETE Journal of Research*, **59** (5):472-478

[4]. Aruna, G. (2014). Performance Analysis of MRC Receiver with Channel Estimation error and CCI in Nakagami-m Fading Channels, *IEEE International Conference on Computational Intelligence and Computing Research*, NCC 2012, IIT Kharagpur, India.

[5]. Aruna, G. (2015). PDF methodology of analysis of L branch equal gain combiner with carrier phase error and CCI over Nakagami-m fading, 2015 International Conference on Computing and Network Communications (CoCoNet), Trivandrum, pp. 505-510.

[6]. Babich, F., and Lombardi, G. (2000). Statistical analysis and characterization of the indoor propagation channel. *IEEE Transaction on Communications.* 48(3), 455-464

[7]. Bessate, A. and El Bouanani, F. (2017). A tight approximate analytical framework for Performance analysis of equal gain combining receiver over independent Weibull fading channels. *J Wireless Com Network*, 3 (2017). https://doi.org/10.1186/s13638-016-0790-2

[8]. Chen, J. I. Z., Liou, C. W. and Yu, C. C. (2010). Error probability analysis of an MC-DS- CDMA system under Weibull fading with a moment-generating function. *Computer and Electrical Engineering,* 36(1), 61-72.

[9]. Cvetkovic, A. M., Djordjevic, G. T. and Stefanovic, M. C. (2011). Performance analysis of dual switched diversity over correlated Weibull fading channels with co-channel interference. *International journal of Communication Systems,* 24(9), 1183-1195.

[10]. Develi, I., and Kabalci, Y. (2016). A comparative simulation study on the performance of LDPC Coded communication systems over Weibull fading channels. *Journal of Applied Research and Technology* (2016), http://dx.doi.org/10.1016/j.jart.2016.04.001

[11]. Galton I. and Weltin-Wu C. (2019). Understanding Phase Error and Jitter: Definitions, Implications, Simulations, and Measurement, in IEEE Transactions on Circuits and Systems I: Regular Papers, **66** (1):1-19.

[12]. Gradshteyn, I.S. and Ryzhik, I.M. (2007). Table of Integrals, Series, and Products Seventh, Edition, Academic Press, San Diego, USA, pp. 1221.

[13]. Hashemi, H. (1993). The indoor radio propagation channel. Proceedings of the IEEE, 81, 943–968.

[14]. Kanellopoulos, S. A., Kourogiorgas, C. I., Panagopoulos, A. D., Livieratos S. N. and G. E. Chatzarakis, (2014). "Channel Model for Satellite Communication Links Above 10GHz Based on Weibull Distribution," in *IEEE Communications Letters*, vol. 18, no. 4, pp. 568-571, April 2014, doi: 10.1109/LCOMM.2014.013114.131950.

[15]. Kapucu, N., Bilim, M., and Develi, I. (2003). Outage Probability analysis of dual-hop decode-and-forward relaying over mixed Rayleigh and generalized Gamma fading channel *Wireless Personal Communications,* 71(2), 947-954.

[16]. Lindsey, W. and Chie, C. (1981). A Survey of Digital Phase-Locked Loops, *Proceedings of IEEE*, **69** (4):410-431.

[17]. Lindsey, W. and Chie C. (1986). Phase Locked Loops, IEEE Press, pp. 234.

[18]. Najib, M. A. and Prabhu, V. K. (2000). Analysis of EGC with partially coherent fadin signals *IEEE Transaction on Vehicular Technology,* **49** (3):783-793.

[19]. Olver, F. W. J., Lozier, D.W., Biosvert, R.F. and Clark C.W. (2010). NIST Handbook of Mathematic Mathematical Functions, Cambridge University Press, New York, USA, pp. 967.

[20]. Quirk, K. J. and Milstein, L. B. (2002). The Effects of Phase Estimation Errors on RAKE Receivers Performance, *IEEE Transaction on Information Theory,* **48** (3):669-682.

[21]. Sagias, N. C., Zogas, D. A., Karagiannidis, G. K., and Tombras, G. S. (2004). Channel capacity and second-order statistics in Weibull fading. IEEE Communications Letters, 8(6), 377–379.

[22]. Simon, M. K. and Alouini, M.S. (2005). Digital Communication over fading channels, John Wiley and Sons. Inc. Hoboken, New Jersey, pp. 937.

[23]. Singh, H. V., Rai, S., Mohan, A., and Singh, S. P. (2011). Robust copyright marking using Weibull distribution. *Computers and Electrical Engineering, 37(5),*714-728

[24]. Singh, R., Soni, S. K., Raw R. S., and Kumar, S. (2016). A New approximate closed-form distribution and performance analysis of a composite Weibull/Log-Normal fading channel

[25]. Stark, H. and Woods, J. (2002). Probability and Random Processes with Applications to Signal Processing, Prentice-Hall, pp. 706.

[26]. Tzeremes, G., and Christodoulou, C. G. (2002). Use of Weibull distribution for detecting outdoor multipath fading. *In Antennas and propagation society international symposium, IEEE, Vol. I* (pp. 232-235).